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Every Body Move: Learning Mathematics Through Embodied Actions

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Every Body Move: Learning Mathematics Through Embodied Actions

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Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May 2012

Acknowledgements

This project was dependent on the contributions of many, and I am incredibly thankful for my advisors, family, and friends for all of their help and support throughout the process of completing this dissertation.

I want to thank my advisor, Taylor Martin, for taking the time to mentor and guide me throughout graduate school. Thank you for continuously challenging me and encouraging me. It has been a privilege to work with you.

The rest of my committee members have also been extremely helpful. Thank you to Leema Berland for all the time spent reading and helping me revise my work, for counseling me, and for always believing in me. Thank you to Catherine Riegle-Crumb for supporting me, particularly early on in my graduate career. To Susan Empson, thank you for continually pushing me with challenging questions and for always having an open door, and to Michael Starbird, thank you for your support and enthusiasm throughout the dissertation process.

There are many other colleagues and friends who have been an important part of my academic community. To Barbara, I cannot think of a better person to go through graduate school with. Thank you for working, studying, writing, thinking, playing, and enjoying life with me the past five years. To Tom, Stephanie, and Pat, the Active Learning Lab would not have been the same without you. You have been incredible people to work with, and moreover, incredible friends. Thank you for being there for me. To the other members of the Active Learning Lab, Bill, Jennifer, Chris, K.C., Cesar, Clara, and others, thank you for listening to many practice presentations, testing my dissertation activities, and giving me feedback on my research. Also, thank you to other current and former Science and Mathematics Education students who mentored and supported me over the years including Stephanie, Vanessa, Sarah, Luz, Shelly, and Margaret.

A special thanks to Matthew Berland for your advice, optimistic view, and continual support of my work. To Jennifer and Lyndee, thank you for all your help keeping paperwork in order and for being good friends.

My dissertation project would not have been possible without the help of Tom, who created technology used in the project, Heidi, who assisted in data collection, Sarah, who assisted with data transcription, and Howard and Melinda.

And finally, my family has been an incredible source of love and support. To my parents, Terry and Cindy, thank you for believing in me. Thank you for teaching me to believe in myself. To my sister and brother-in-law, Lauren and Jordan, thank you for your love and encouragement and for giving me feedback from the teaching world. And to my husband, Steve, thank you for your love, your unending kindness, and unwavering support.

Every Body Move: Learning Mathematics Through Embodied Actions

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The University of Texas at Austin, 2012

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Giving students opportunities to ground mathematical concepts in physical activity has potential to develop conceptual understanding. This study examines the role direct embodiment, an instructional strategy in which students act out concepts, plays in learning mathematics. I compared two conditions of high school geometry students learning about similarity. The embodied condition participated in eight direct embodiment activities in which the students represented mathematical concepts and explored them through their movements. The observer condition participated in eight similar activities that did not involve physical activity.

The students in the embodied condition had greater learning gains on a pre- and post-test, and those gains were driven by larger increases in conceptual understanding. There were also differences in the way the two conditions remembered the activities. On a survey given at the end of the unit, students in the embodied condition were more likely to write about the activities from a first person point-of-view, indicating that they had likely adopted a first person viewpoint during the activities. The embodied condition was also more likely to switch points-of-view when writing about the activities, indicating

that they had likely translated among multiple viewpoints during the activities. This suggests translating between viewpoints is one mechanism for learning through direct embodiment.

Students in the embodied condition also wrote more about the activities, which suggests that they remembered more about their experiences. Their survey responses included more mathematical and non-mathematical details than the responses from students in the observer condition.

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Introduction

Embodied cognition theory has made several advances in the last few decades; however, much less research has considered how the theory might be applied to instruction. This dissertation closely examines direct embodiment, one instructional technique that is based off of an understanding that cognition is embodied. It is arranged as three standalone articles. Section 1, titled "Direct Embodiment and Implications for Instruction," seeks to describe the theoretical foundations and research evidence supporting direct embodiment. Section 2, titled "Learning Mathematics Through Acting It Out," describes a research study that investigates what kind of impact direct embodiment has on student learning, and finally, Section 3, titled "Switching Viewpoints While Learning Mathematics Through Direct Embodiment Activities," examines the differences in the learning process for students participating in direct embodiment activities compared to students learning through more passive activities.

SECTION 1: DIRECT EMBODIMENT AND IMPLICATIONS FOR INSTRUCTION

Outside of school contexts, we learn through a variety of strategies including everything from experimenting and testing out ideas to reading Wikipedia or watching online videos. When we learn on our own, we naturally gravitate towards a strategy that fits the content we are learning. Despite this natural variation in our learning strategies, lecture remains the dominant mode of instruction in schools, and students are given little control or space to try out their own ideas.

This is not a new problem. Many years ago, Dewey (1926) asked, "Why is it, in spite of the fact that teaching by pouring in, learning by passive absorption, are universally condemned, that they are still so entrenched in practice?" (p. 46). Even with numerous reform efforts over the years, direct instruction and individual learning remain ingrained in schools (Stigler, Gallimore, & Hiebert, 2000). Part of the video component of the Third International Mathematics and Science Study (Stigler et al., 2000) examined a sample of 81 eighth grade mathematics classes in the United States and found that most of the classes that were observed reliably followed a particular pattern of events: the class reviewed previous material through a "warm-up" or checked homework, the teacher modeled a procedure to solve problems, the students practiced the steps individually, the class checked the practice problems, and the teacher assigned homework. Lather, rinse, repeat.

Throughout all of these regular math class activities, the students remained seated at their desks, and there were few opportunities for them to actively engage in the content.

I am interested in how physical activity can help students think about and learn mathematics—an area that has not been thoroughly investigated. In western culture, the dominant image of intellectual contemplation is someone sitting, motionless, perhaps with a furrowed brow. The fact that the most famous work of art portraying a man thinking is a *statue* made of bronze and marble is quite telling. This illustrates the separation in our conception of what the mind and the body do. The mind is for thinking, and the body is its support system. This carries over into schools, and many of our instructional practices are informed by this philosophy.

Embodied cognition theory has begun to erase the mind/body separation (Barsalou, 2008; Barsalou, Niedenthal, Barby, & Ruppert, 2003; Gibbs, 2005; Glenberg, 1997; Lakoff & Johnson, 1999; Wilson, 2002). According to this theory, in order to make sense of the world, our brains use re-activations of past sensorimotor states. When we acquire new information, our brain stores that original neural state, which is activated again whenever we recall that information. This means our cognitive processes are intertwined and somewhat indistinguishable from our embodied experiences. Furthermore, this process applies to all cognitive activity, from thinking and problem solving all the way out to conceptual development (Barsalou, 2008; Glenberg, 1997; Wilson, 2002).

While cognitive science has made many advances over the past three decades in the field of embodied cognition, there is little research on how to apply this theory to instruction (Glenberg, 2008). We must consider the following question: What instructional practices will most likely tap the affordances of students' embodied cognitive experiences? This is not a question of what kinds of activities will help students produce embodied thought. Whether you are seated, reading and making sense of this text or actively learning mathematics with manipulatives, your thoughts draw upon embodied experiences. Rather, this question means finding ways to take advantage of the connection between the way we perceive information and the way we think when designing learning experiences.

In this section I expand on direct embodiment, an instructional strategy described by Fadjo, Lu, and Black (2009) that capitalizes on the mind-body connection. I propose four qualities of instructional activities that utilize direct embodiment, and I give research support for and examples that illustrate each. I also attempt to distinguish direct embodiment from other constructivist instructional strategies. At the end of the paper, I will delve briefly into imagined embodiment, another instructional strategy based on embodied cognition theory.

Direct Embodiment

Fadjo et al. (2009) define direct embodiment as "the use of the human body to act out a pre-defined scenario" (p. 4041). In their study, they worked with elementary school students in an after school program who were learning to program video games in Scratch. In one condition, students used direct embodiment to help them learn new programming commands. The teacher chose volunteers to use their bodies to physically act out pre-defined programming scripts as if they were characters in a video game. A control group imagined acting out the scenarios. The programming scripts for both conditions contained conditional statements. Then, the students designed programs on computers and had the opportunity to implement the commands they had learned. The students who learned through direct embodiment wrote more complex conditional statements than a control group who imagined acting out the pre-defined programming scripts.

Another common example of learning through direct embodiment comes from mathematics education research studying students learning about rate of change (Nemirovsky, Tierney, & Wright, 1998; Noble, Nemirovsky, Wright, & Cornelia, 2001; Wright, 2001). In exploring ideas about rate of change, students can directly embody distance time-graphs. For example, if a student were to directly embody the graph in Figure 1.1, the student's actions might look something like that depicted in Figure 1.2. Box A in Figure 1.2 shows the student starting at distance 0. In Box B, the student walks two feet in two seconds. Box C shows the student standing still for three seconds. Finally, in Box D the student walks four more feet in three seconds.

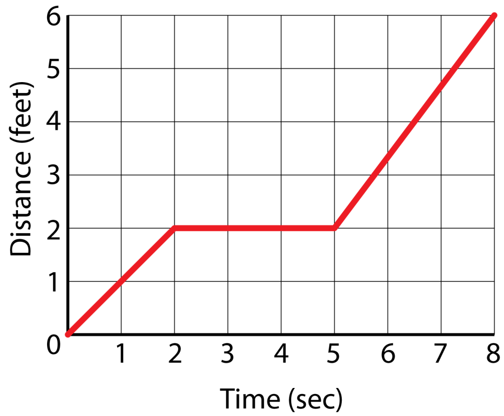


Figure 1.1. Distance-time graph.

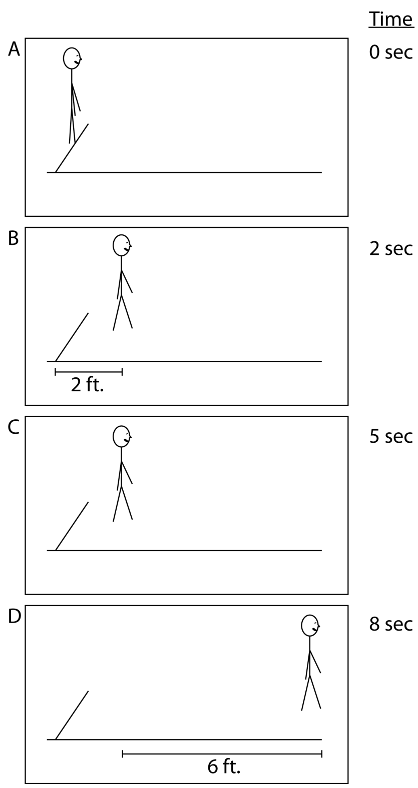


Figure 1.2. Directly embodying the distance-time graph in Figure 1.1.

Learning through direct embodiment activities gives students a physical experience that they can relate directly to a mathematical representation. Students become a part of the mathematics, and they are able to learn through their movements. I call this "learning by being" and "learning through moving." Later in this paper, I will expand on these ideas and describe how direct embodiment activities incorporate both of them.

Designing instructional activities to help students learn mathematics through direct embodiment is not necessarily easy. Since mathematics can be very abstract, determining how students can directly embody some concepts is not always immediately obvious, and it may not be possible in every case. These kinds of activities can look very different depending on the content, the students, and the resources available; however, I have identified four main qualities that direct embodiment activities share and will expand upon each. In direct embodiment activities:

1. Students create a physical representation of mathematics.
2. Students' actions are mathematically meaningful.
3. Students' actions allow them to test their ideas.
4. Students take different viewpoints from which to examine mathematics.

These four aspects can serve as a guide for developing direct embodiment activities, but they by no means encompass all the necessary elements of this form of instruction.

1. Students create a physical representation of mathematics.

We learn new concepts by drawing on our past experiences (Bergen & Feldman, 2008), and the nature of mathematics is such that much of it is composed of mental abstractions that cannot be perceived or experienced (Nuñez, 2008). For example, no one can experience infinity. Depending upon your level of interest, you might feel as if reading this paper will take an infinite amount of time, but in reality, you will stop reading it at some point. You will not actually experience reading this paper for all eternity.

Mathematical concepts may be more difficult to learn if we do not have past experiences that we can use to help use make sense of them (Abrahamson & Howison, 2010; Han, Black, & Hallman, 2009). If instruction gives students physical experiences that connect to mathematics concepts, it might make those concepts easier to learn and understand. Following Barsalou's (1999) proposed Perceptual Symbol System, the physical experience of direct embodiment would create a perceptual symbol of the concept. Students would understand what the math *feels* like and be able to recall that experience when later when thinking about mathematics in more abstract contexts.

Evidence from Brain Research. Brain research provides support for the theory that we access ideas through perceptual symbols (Barsalou, 1999). Pulvermüller, Härle, and Hummel (2001) used high-resolution electroencephalogram (EEG) recordings to detect electrical activity in the brain of study participants who were visually presented with words that were verbs referring to actions. They found that when a participant saw a verb, the motor and pre-motor cortex areas of the brain that were associated with the parts of the body that performed the verb were activated. For instance, if a participant saw the word "smile," the areas of the brain controlling the mouth were activated.

Similarly, Stanfield and Zwaan (2001) examined the relationship between perceptual symbols and changes in the visual images to which they referred. Study participants read a sentence such as "John put the pencil in the cup." Then they were shown an image and had to determine whether or not it was mentioned in the sentence. It took participants longer to perform the task if the image was oriented in a way that did not corresponded with the sentence (e.g., a pencil oriented horizontally) and less time if it was oriented the same way (e.g. a pencil oriented vertically). A mental rotation task showed that this was the case for all participants regardless of their spatial ability. In other words, everyone creates and uses perceptual symbols, not just those with good spatial skills. This also suggests differences in the perceptual symbols reflect the differences in the visual images.

Evidence from Language Research. Glenberg and Robertson (1999, 2000) explain how the theory of embodied cognition applies to language comprehension through their Indexical Hypothesis. Glenberg, Gutierrez, Levin, Japuntich, and Kaschak (2004) found that first and second grade children improved their reading comprehension if they spent time manipulating toys to mimic the actions described in the text. For example, a child might read a text about a farm scene, and at the end of certain sentences, the child would be prompted to manipulate farm animal toys to produce the actions the were described in the text. Manipulating the toys allowed the child to index the toy to the perceptual symbol or embodied representation of the word that the toy represented. In learning through direct embodiment, a related form of indexing occurs. Students create a physical representation that that they can index to a mathematical concept.

Glenberg, Willford, Gibson, Goldberg, and Zhu (2011) found evidence that a similar reading comprehension intervention improved students' performance on mathematics story problems. Students used a mouse to manipulate images of toys on a computer screen to simulate the story problems as part of an intervention. A control group saw the images of the toys on the screen but could not manipulate them. The experimental group scored answered more problems correctly than a control condition. Since the intervention was designed to improve reading comprehension, the researchers posit that this difference was due to the experimental condition understanding the problem scenario better. The intervention was not designed to help children index mathematical concepts (such as counting) in the story, and the children were not performing mathematical actions with the toys.

There is a distinction between students comprehending a problem situation (which Glenberg's intervention helps with) and understanding mathematical relationships (which direct embodiment can help with). Direct embodiment activities are designed to help students index mathematical concepts to the their physical actions. Their actions create a perceptual representation of the mathematics concept for the student to recall later, and this is important for students to be able to understand and solve problems.

Evidence in mathematics. There is evidence that we naturally index some mathematics concepts with perceptual representations. One common way this occurs is through counting. Many children learn to count by using their fingers. Because of this, Andres, Seron, and Olivier (2007) thought there might be a relationship between the hand muscles and counting tasks. They found increased activity in hand motor circuits in adults who were undertaking counting tasks. They found no increase in corticospinal excitability in arm and foot muscles for the same task. This means that the hand muscles are specifically involved in counting tasks while other muscles are not.

Sato, Cattaneo, Rizzolatti, and Gallese (2007) found a similar relationship in a non-counting, numerical task. They presented study participants with a number (1-9) and asked them to determine if they were even or odd. They found an increase in corticospinal excitability in the right hand during these tasks.

Not only do these relationships occur naturally, but interventions can help children to develop embodied representations of mathematics. Ramani and Siegler (2008) found that playing board games can help children develop a conceptual model of the number line. They compared low-income pre-school children playing a linear board game labeled with numbers to playing an identical version of the game with different colored squares instead of number labels. Their results show that playing the number board game for one hour increased performance on numerical tasks more than playing the color board game, and the gains in performance remained after nine weeks.

Concepts direct embodiment can teach. Lakoff & Nuñez (2000) propose that all of mathematics can be explained by layers of conceptual metaphors that are grounded in bodily experiences. For example, they suggest that we understand arithmetic through the metaphor "Arithmetic is Motion Along a Path." Teachers can tap into this metaphor by creating activities in which students directly embody arithmetic by walking on a line on the floor. A student might read the problem, "Stephen walks 8 feet towards the front of the room. He stops to tie his shoe. Then he walks 7 more feet in the same direction. How far is Stephen from where he began?" and then solve it through direct embodiment.

As the student walks eight units and then walks an additional 7 more units, the student is creating a physical experience that can tie back to addition.

Still, the research on this type of instruction is in its infancy. Direct embodiment activities have been developed to help students learn a variety of mathematics concepts: geometric proofs (Srisurichan et al., under review), probability (Abrahamson, 2004), proportions (Abrahamson & Howison, 2010), rate of change (Nemirovsky et al., 1998; Noble et al. 2001; Wright, 2001), patterns (Petrick, 2011), interpreting graphs (Gerofsky, 2010), and arithmetic (Goldin-Meadow, Cook, & Mitchell, 2009). However, direct embodiment of more abstract, higher level mathematics has not been investigated.

2. Students' actions are mathematically meaningful.

A large amount of research over the past few decades has focused on "learning by doing" (e.g., Gibbs, 1998; Schank, Berman, & Macpherson, 1999; Roussou, 2004; Smart & Csapo, 2007; Smith, 1998). The emphasis of these studies is on students becoming active participants in their learning. There are many forms of "learning by doing" instruction that share similarities to learning through direct embodiment. I will explore those similarities and then expand on why in direct embodiment activities students "learn by being" rather than "learn by doing," and I will look closely at gesture research, which provides some important insights into "learning by being."

Learning by doing. In active learning, students do more than just listen to direct instruction (Bonwell & Eison, 1991; Smart & Csapo, 2007). They read, write, and discuss topics. Bonwell and Eison (1991) define active learning as "anything that involves students in doing things and thinking about the things they are doing" (p. 2). This is similar to direct embodiment instruction in that it is student-centered.

In project-based learning, students learn by doing real-world projects (Barron et al., 1998; Blumenfeld et al., 1991). These projects are most successful at fostering learning when students draw relationships between the activity and the content (Barron et al., 1998). Petrosino (1998) found that the way the project is framed can change the learning outcomes for students. These two findings are important lessons for direct embodiment activities. The connection between students' actions and the mathematics must be clear.

Like in project-based learning, goal-based scenarios (Schank, 1996; Schank et al., 1999) put students in a real-life position where they are motivated to learn in order to solve authentic problems. For Schank (1996), learning by doing in goal-based scenarios involves forming hypotheses and testing ideas, which is an important part of direct embodiment activities that will be explored in the next section.

Experiential learning is "the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience" (Kolb, 1984, p.41). In experiential learning theory, students learn through concrete experiences, reflective observation, abstract conceptualization, and active experimentation (Kolb, Boyatizis, and Mainemelis, 2000).

In all of these forms of instruction, students *do* mathematics. "Learning by doing" involves much more than just passively absorbing information; students are active. However, the type of activity in "learning by doing" is different than in direct embodiment activities.

Learning by being. In direct embodiment activities, students "learn by being." As described previously, during these activities students physically represents mathematics concepts. They *are* the mathematics, and their actions have mathematical meaning. For example, if a student is directly embodying an angle by holding her arms out to form the sides (See Figure 1.3), in many senses, the student *is* the angle.

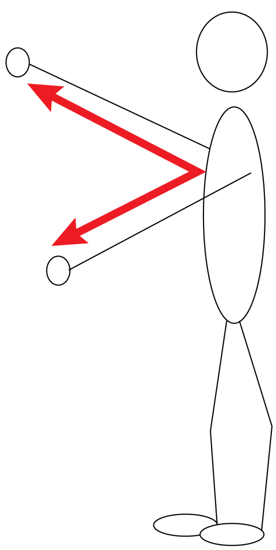


Figure 1.3. Directly embodying an angle.

There is a blurring of the line between student and the math. This is what Nemirovsky et al. (1998) refer to as fusion. In fusion the mathematical symbol (the angle) and the event or situation (the student holding her arms straight out to one side) merge, and the way students talk and think about the two are indistinguishable.

Once fusion has occurred between the student and the mathematics concept at hand, the actions of the student, or what the student *does*, has mathematical meaning. The student might rotate her body and observe that despite the rotation, the angle measure between her arms remains the same, or the student might add another angle to the angle she has formed by widening her arms.

Mathematically meaningful gestures. Mathematically meaningful actions are important for learning. Goldin-Meadow, Kim, and Singer (1999) examined 8 teachers individually instructing 49 students about mathematical equivalence. They looked for how teachers use gesture in their instruction and what the effects of these gestures are on the student.

As the teachers were instructing the students, they conveyed a specific strategy in speech that was not accompanied by gesture about 36% of the time. About 39% of the time, the teacher conveyed a specific strategy in speech that was accompanied by a gesture conveying the same strategy. About 19% of the time, the teachers showed the students a strategy through gesture that did not match what they were saying in speech. And finally, occasionally, teachers produced strategies in gesture that were not accompanied by speech.

The students were less likely to reproduce the teacher's verbal strategy if it was given with a mismatched gesture strategy than when it was produced with no gesture at all. Also, they were more likely to reproduce the teacher's verbal strategy if it was given with a matched gesture strategy. In other words, when the teachers' gestures had mathematical meaning reinforcing their verbal message, the students were more likely to take up the idea.

Alibali, Flevares, and Goldin-Meadow (1997) found that students' gestures can convey important information for teachers about their mathematical reasoning. Students often reveal information about their mathematical thinking in gesture before speech (Church & Goldin-Meadow, 1986). When a student's gesture and speech do not match, this is called the transitional knowledge state, and Church and Goldin-Meadow propose that when a student is in this state, she is most receptive to instruction.

In direct embodiment activities, students' actions have mathematical meaning. It is possible then that teachers could make interpretations about a student's level of understanding by watching how she acts during a direct embodiment activity. The teacher may also be able to identify students in the transitional knowledge state, who display actions that are beginning to reveal a mathematically correct strategy but speech that is rooted in a different strategy. Once identified, the teacher may be able to provide further instruction that will lead those students to a deeper level of conceptual understanding.

3. Students' actions allow them to test their ideas.

Constructivist theories of learning have long championed instruction that enables students to make sense of material by forming and testing hypotheses. Piaget (1973) believed that children's actions play an important role in their development. Children's ideas about the world are continually evolving, and conceptual change arises from their experience interacting with the world (Ackermann, 2004). Children's actions are important because they "learn *through* moving." This is different than "learning and moving."

Learning and moving. With the popularity of Gardner's (1983) multiple intelligence theory, many learning approaches have appeared that incorporate both kinesthetic activity and learning. Some kinesthetic learning activities fall under the umbrella of direct embodiment (e.g., Barnes & Jaqua, 2011; Touval & Westreich, 2003). However other activities involve students learning while moving in different ways such as crawling, clapping, or stretching (e.g., Hyatt, 2007; Franco & Dauler, 2000). These kinds of activities are great for engaging students, but the movements students make are not related to the content they are learning and do not allow students to use their movements to make and test mathematical conjectures. In contrast, when students learn *through* moving, their movements reflect their thinking, and moving allows them to experiment with different ideas and strategies.

Learning through moving. There are multiple ideas about how students can learn through their movements or actions. One theory on action is that students' movements focus their attention on the information displayed as a result of their actions (Goldin-Meadow et al., 2009). In this way of thinking, just doing or repeating someone's actions could help students learn.

Another idea is that action could be part of a learning cycle. When a student is moving as part of the learning process, his or her actions are purposeful and, as described above, meaningful. The actions that a student takes when she is directly embodying a problem space change the problem space, which in turn change what the student perceives. The new perceptions change the student's thought processes, which consequently change the actions a student takes. This process is similar to the theory of physically distributed learning (Martin, 2008; Martin, Svhila, & Smith, in press).

Ordinarily, when we think of perception (whether visual, tactile, etc.), we think of a passive process. Our bodies perceive information that is sent to the brain, and sometimes the brain chooses to act on this information. There is an emerging theory on perception in which all of the senses work together with the body to perceive and interact with the environment (Nemirovsky & Borba, 2004).

In direct embodiment, students are actively moving and perceiving information. What they perceive and how they perceive it changes how they think, and their new thoughts inform new actions.

Affordances and direct embodiment. When a student directly embodies a mathematical concept and has begun to see herself as that concept, a whole new set of affordances become available to the student (Gibbs, 1977). For example, normally for a student walking is just walking. It allows students to get to class on time and serves as a form of exercise, but when a student is directly embodying a distance-time graph, as described earlier, walking opens up a whole new set of possibilities.

For a student learning about distance-time graphs through interacting with a motion-detector, the direction a student walks affects the distance-time graph—the closer the student walks to the motion detector, the smaller the y-value on the graph, and the further the student walks away from the motion detection, the larger the y-value. The speed the student walks affects the slope of the graph—the faster the pace, the steeper the slope, the slower the pace the flatter the slope. Using these new tools, the student can use her actions to try out different ideas and learn more about the graphs.

Taking action and learning. Research on distributed cognition has shown that we often off-load cognitive work onto the environment. Kirsch (1995) found that people who used their hands to rearrange a set of coins in strategic ways could count them significantly faster and more accurately than those who were not allowed to manipulate the coins. Kirsh and Maglio (1994) found that in the game of Tetris, players rotated and translated falling objects in order to make the decision of where to place them easier.

Martin (2008) describes "fussing" as a way that students' can increase variability in their actions, which can get them out of unproductive patterns of movements. In direct embodiment activities, instead of fussing with manipulatives, students can "fuss" with their movements. They can try out a variety of ideas, and this will hopefully move them towards using the more successful strategies.

4. Students take different viewpoints from which to examine mathematics.

As will be described in much more depth in Section 3, learning mathematics through direct embodiment offers students an opportunity to take a different viewpoint when thinking about mathematics. A viewpoint is a "locus of consciousness for a model of the world" (Parrill, 2009, p. 272). Wright (2001) describes how a student takes different viewpoints while solving rate of change problems. In the problem, two people begin at a starting line and follow different kinds of motion (walking slowly, running, etc.) for different amounts of time. The student must decide who will reach the finish line first. Wright tells how when the student first reasons about the problem, she answers incorrectly, but when she takes a viewpoint from within the problem, it helps her to imagine other possibilities. This leads her to more advanced mathematical thinking. For the student, directly embodying the problem afforded translating to a first person, or participant, viewpoint.

It is possible to take a first person viewpoint without direct embodiment. For example, take a moment to think about how many doors are in your home. How did you come up with your answer? To answer this type of problem, most people mentally insert themselves into their homes and imagine walking through it (Bergen & Feldman, 2008).

Smith (1991) asked study participants to sketch a drawing of the home they lived in at age 5. To do this, participants would alternate viewpoints. They would imagine a first person viewpoint inside their home, and then translate to a third person viewpoint to create the sketch. Taking the first person viewpoint enabled the participants to gather information they could not get from the third person viewpoint. For example, one participant used a first person viewpoint to help him remember if there was space to walk from one room to another by moving behind the couch. The study participants translated seamlessly between viewpoints to help them recreate the sketch.

In Section 3, I will present evidence that direct embodiment activities afford switching between viewpoints. For many mathematics topics, it may not always be obvious to students how they can use imagination to translate to a first person viewpoint—especially for more abstract concepts such as ratio and proportional reasoning. A well-designed direct embodiment activity may offer a greater affordance for students to take a first person viewpoint through direct embodiment. Class discussions and interactions with other students could then help students translate information gained from a first person to a third person viewpoint.

Imagined Embodiment

As previously noted, it is not necessary to physically enact a problem in order to take a first person viewpoint. Brain research shows that imagining movements produces the same corticospinal excitability patterns as making the actual movements (Fadiga, 1999). Therefore, it may be possible for students to imagine embodying mathematics concepts and gain the same benefit as if they were directly embodying them.

I hypothesize that imagined embodiment is useful when students already have some background knowledge and experience with the subject, and it is an important link between direct embodiment and more abstract reasoning. When introducing a new concept to students, it is likely that starting with direct embodiment activities is important for students to have a chance to ground mathematics concepts in their physical actions. Without this step, it may be more difficult if not impossible for students to make connections between imagined actions and mathematics. Once students have mapped concepts to their actions, imagined embodiment could be a useful and important instructional technique.

Glenberg et al. (2004) investigated reading comprehension and found that students who manipulated toys to replicate actions in a text and later imagined manipulating toys to replicate the text had greater comprehension than a control group who re-read the text. The imagined manipulation occurred after the students were able to successfully index the words of the text to the actual toy manipulations. Had the children only imagined manipulating the toys and never physically manipulated them, it is unknown whether or not there would have been gains in comprehension.

Fadjo et al. (2009) compared students learning through only direct embodiment or only imagined embodiment and found that the students who in the imagined embodiment condition implemented the new programming commands they learned less frequently than the direct embodiment condition. In this case the students were imagining acting out commands that they had never acted out before. Had the students started by learning the commands through directly embodiment and then moved from there to imagined embodiment, it is possible that they would have been more successful at implementing the new commands in their programs.

Gesture research has found a similar phenomenon. Alibali and Nathan (2007) found that teachers use gestures more often when they introduced new and unfamiliar material. Their gestures grounded their language with real-world, physical referents, and they emphasized relationships between mathematics concepts. This finding supports the hypothesis that direct embodiment activities are more effective when introducing new material than imagined embodiment. However, more research in the area of imagined embodiment is needed to determine how it can best be implemented as an instructional strategy.

Conclusion

I have outlined four qualities of direct embodiment activities which are important to consider when designing instruction, and I have distinguished between learning through direct embodiment, which encompasses "learning by being" and "learning through moving," from other learning activities which include "learning by doing" and "learning and moving." As researchers continue to look deeper at the role direct embodiment activities play in learning, the ideas presented here will no doubt evolve.

In particular, it will be interesting to take a closer look at how students' viewpoints change during direct embodiment activities and what impact that has on learning. It will also be interesting to see how students respond to direct embodiment activities that teach more abstract concepts, and finally there are many unanswered questions about the role of imagined embodiment.

SECTION 2: LEARNING MATHEMATICS THROUGH ACTING IT OUT

Introduction

The intuition of teachers, parents, researchers – basically anyone observing how people learn - is that physical activity plays a key role in our learning. In the area of mathematics for example, we often see children acting out a problem to help them solve it or teenagers in a math class gesturing to explain a claim about a graph. There is something about physically enacting a problem or gesturing while speaking that seems to enhance the learning process. However, mathematics is widely taught through more traditional instructional methods with an emphasis on whole class instruction and the use of worksheets and textbooks (Riley, 2003). These teaching practices rarely give students opportunities to physically act out mathematics problems.

In traditional amodal theories of cognition no relationship exists between mental representations and the perceptual states that produce them (Pylyshyn, 1984). There is growing research support for an embodied view of cognition that relies on modality-specific perceptual states (Barsalou, 1999; Barsalou, 2008). In embodied cognition theory, the brain, body, and environment work together to help us make sense of the world (Glenberg, 1997; Lakoff & Johnson, 1999; Wilson, 2002). Because of this, it is becoming more important to examine the role of the body in learning.

On one hand, mathematics instruction which helps students make connections between their own physical experiences and abstract concepts (referred to as direct embodiment, see Fadjo et al., 2009) could help students develop conceptual understanding by giving them a concrete, personal experience which they can feel and relate to (Abrahamson & Howison, 2010; Gerofsky, 2010; Han et al., 2009). For example, if a student studies rate of change by acting out a distance-time graph, the student can associate the physical feeling of accelerating to the increasing slope of the graph.

On the other hand, it could be that what is important is observing representations of a concept. For example, it may be enough to watch a video of a car accelerating and draw connections between that and an accompanying graph. In other words, the physical experience of embodying mathematics concepts might not offer any added benefit towards learning.

There is currently not enough evidence to know exactly what role physical experiences play in conceptual development, therefore, I ask: Does learning through direct embodiment (i.e. activities in which students act out math concepts) have a measureable impact on student learning, and how does it compare to more passive instruction? I tested the hypothesis that using one's body to enact mathematics concepts may result in greater conceptual development than observing or drawing abstract representations of mathematics concepts.

An issue that arises in this idea of "testing" embodied learning is that I do not posit a learning environment that is not embodied. Following the theoretical claims of embodied cognition, all cognitive processes including problem solving, remembering, imagining, and introspection are embodied (Gibbs, 2005; Barsalou, 2008; Wilson, 2002).

However, I was interested in comparing activities that were more and less likely to engage the benefits direct embodiment could provide while learning novel mathematics content. In other words, I maximized the chance of tapping into the affordances of direct embodiment for learning something new.

In my study, high school geometry students participated in a two-week unit on similarity in which they studied ratio and proportion, similar figures, and real world applications of these concepts under one of two conditions (embodied and observer). One hundred sixty-two geometry students ages 14-19 years old and four geometry teachers from an urban high school participated in the experiment. The teachers taught a combined 14 sections of geometry. I randomly assigned half of each teacher's classes to the embodied condition and the other half to the observer condition. This minimized any teacher effects since each teacher taught both conditions.

During the course of the unit, the students took part in eight small group learning activities. Both conditions had similarly structured activities with identical content. I wanted to isolate any effect direct embodiment might have on learning, so the classes in the embodied condition physically acted out mathematics concepts, while the classes in the observer condition watched videos or drew pictures of the same concepts. For example, in an activity on similar triangles, the students in the embodied condition worked in groups of three. Each student physically represented a vertex of a triangle and held retractable measuring tapes, which denoted the sides of the triangle. The groups worked together and moved their bodies to create triangles of different sizes. Then they measured the sides and angles, and developed a rule for creating triangles of the same shape. The teacher then compared the students' rules with the definition of similarity. In contrast, the observer condition completed the same activity working in groups of three and had the same objectives, but instead of creating the triangles with their bodies, they drew them on paper.

The activities for both conditions included similar tasks as well as the same discussion questions. An outside mathematics expert verified that the content of the activities for both conditions was equivalent, and education experts ensured that the quality of instruction was equivalent.

I administered a 17-question pre- and post-test before and after the unit. The test was designed to measure procedural and conceptual understanding and included items from previous research studies (Lamon, 1993; Misailidou & Williams, 2003), released items from the National Assessment of Educational Progress, and school assessments.

Results

The students in both conditions had significant learning gains from pre- to post-test, $F(1, 160) = 481.6$, $p < 0.001$, meaning that all students learned the material. There was a significant condition x test-time interaction, $F(1, 160) = 4.4$, $p < 0.05$ (See Figure 2.1). This means that while there were no differences between the two conditions' pre-test scores, the students in the embodied condition had significantly greater learning gains from pre-to post-test than the students in the observer condition. Both conditions learned, but the students in the embodied condition learned more.

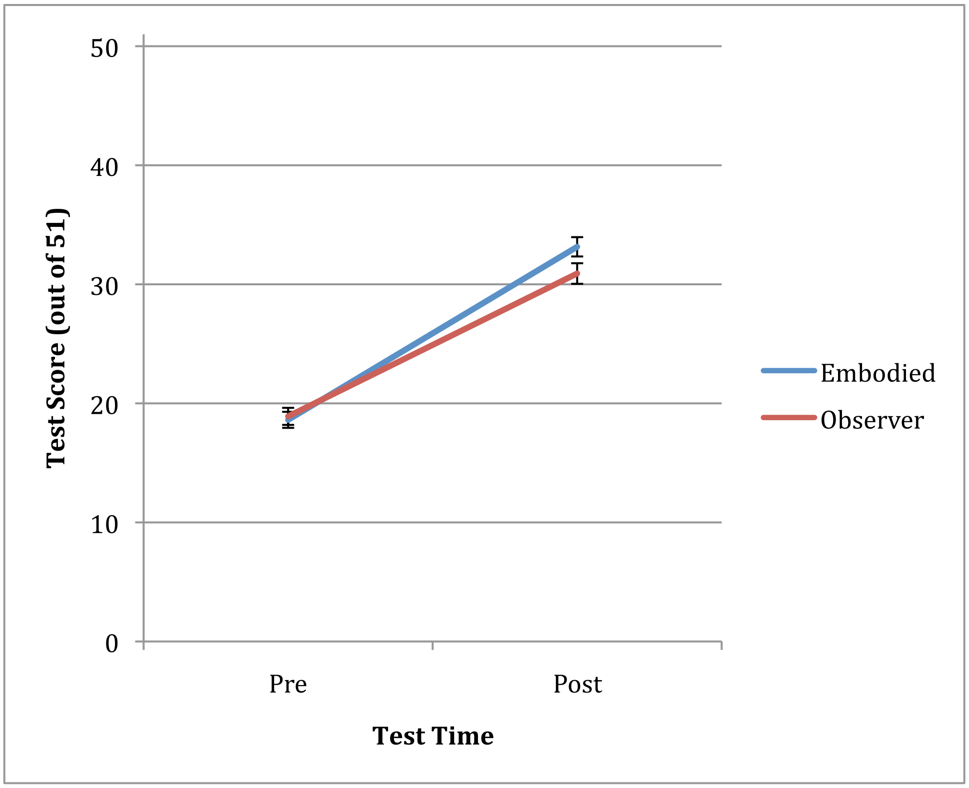


Figure 2.1. Pre- and post-test scores by condition.

To further explore the differences between the conceptual and procedural test items, I performed a doubly multivariate repeated-measures analysis (Tabachnick & Fidell, 1996) with condition (embodied, observer) as the between subjects factor, time (pre, post) as the repeated-measures factor, and both conceptual and procedural scores as the two within-subjects measures. There was a significant overall condition x time interaction for the two within-subjects measures, $F(2, 159) = 6.8$, $p < 0.01$. Univariate analyses revealed that the variable contributing to the overall significance of interaction was the measure of conceptual understanding, $F(1, 160) = 12.4$, $p < 0.01$ (See Figure 2.2).

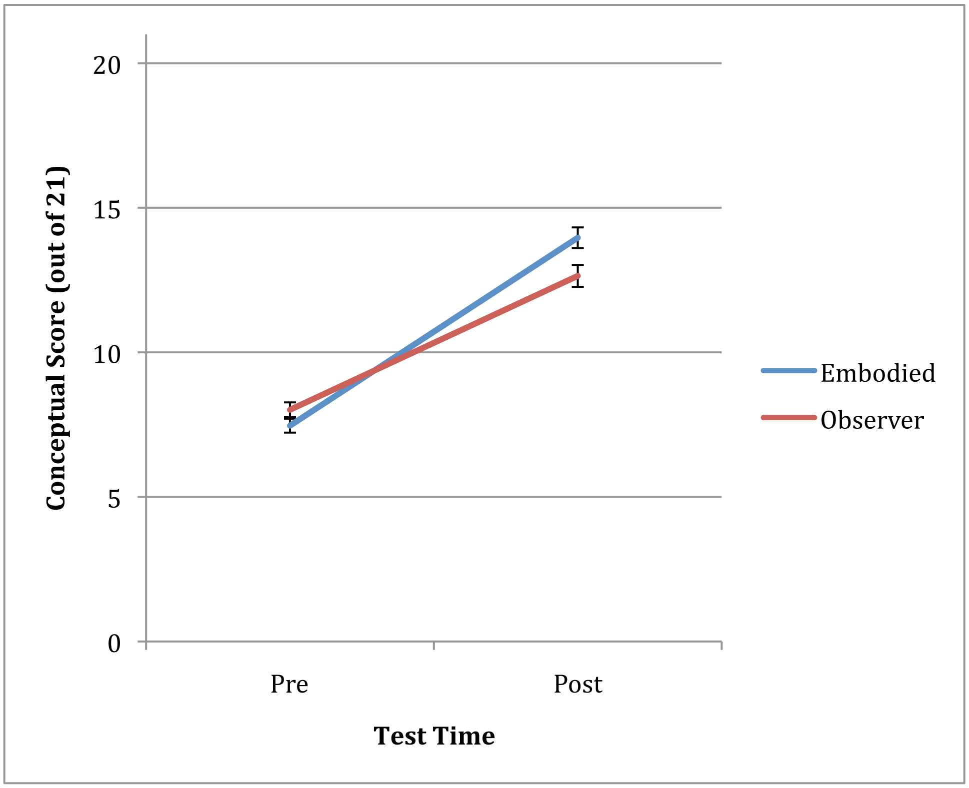


Figure 2.2. Conceptual pre- and post-test scores by condition.

There were no significant differences in procedural understanding by condition (See Figure 2.3).

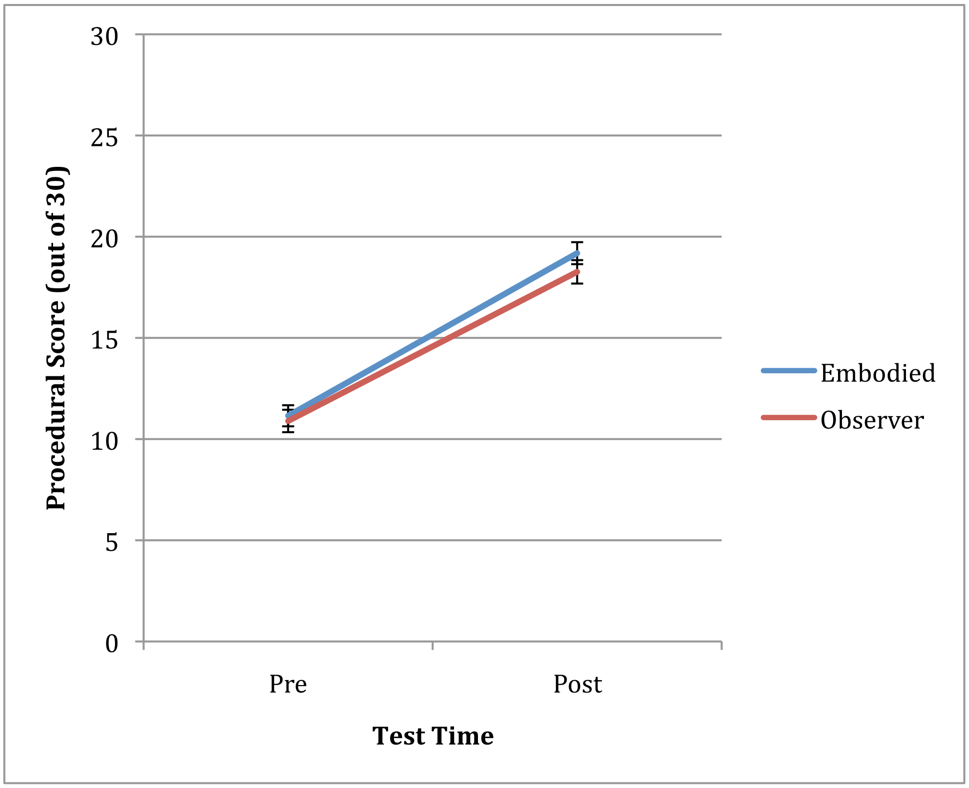


Figure 2.3. Procedural pre- and post-test scores by condition.

This means that for the procedural items on the test the two groups of students performed similarly on both the pre- and the post-test, but for the conceptual items, the students in the embodied condition had significantly greater learning gains. The difference in the learning gains on the conceptual items accounts for the significant interaction effect in overall test scores.

Discussion

These findings suggest that giving high school students opportunities to embody mathematics concepts and physically enact them may help them learn by deepening their conceptual understanding. It will be interesting to examine whether or not direct embodiment has the same effect on learning for other mathematics concepts and for different age groups of students.

Steering students from thinking about mathematics as a set of rules that must be memorized regardless of if they make sense toward developing a deep conceptual understanding of mathematics has been an important but elusive goal of mathematics educators and researchers for many years. This study suggests that instruction that gives students the opportunity to directly embody mathematical ideas can increase conceptual understanding at a greater rate than when students do not have these opportunities, while showing equal gains in procedural understanding. While I am not suggesting that students should act out every math problem they work, I do think employing direct embodiment as an instructional strategy for difficult to understand, abstract concepts could be useful. Direct embodiment might be one of the keys to making math make sense.

Supplementary Materials

Methods

Participants. One hundred sixty-two high school geometry students ages 14-19 years old and four geometry teachers from the same urban high school participated in this experiment. They did not receive compensation for participation. The teachers taught a combined 14 sections of geometry. I randomly assigned half of each teacher's classes to the embodied condition and the other half to the observer condition (See Table 2.1). This minimized any teacher effects since each teacher taught both conditions.

Table 2.1. Number of classes per condition by teacher.

Teacher	Embodied	Observer
A	2	3
B	2	2
C	2	1
D	1	1
Total	7	7

Assessment Instruments

To measure student achievement, I gave a pre- and post-test at the beginning and end of the unit (See Appendix A). The 17-item test was comprised of problems adapted from other research studies (Lamon, 1993; Misailidou & Williams, 2003), released NAEP items, and from school assessments. It was designed to measure both conceptual and procedural understanding following the definitions given by Rittle-Johnson, Siegler, and Alibali (2001). Seven items were classified as conceptual items, and ten items were classified as procedural items.

Procedure

The teachers received 13 hours of professional development conducted by one of the researchers. The professional development centered on delivering the curriculum and focused on instructional strategies such as leading class discussions. The teachers were instructed to follow the lesson plans as closely as possible and to minimize the differences between the two conditions with the exception of the physical activity. For the embodied condition, the teachers were instructed to use gestures and actions to convey their ideas, but for the observer condition, they were asked to refrain as much as possible from using gestures and actions as they taught.

The students in both conditions were told that they would be participating in an experiment in which their teacher was trying a new kind of instruction. The instruction for both conditions was different than the normal curriculum in that it included many more conceptual development activities, more group work, and more small group and whole class discussion than their normal classes. In other words, both conditions felt as if they were experiencing something new and different from their normal instruction.

The unit included 9 class days over the course of two weeks. On Day 1, students took the pre-test. Days 2 – 7 were instructional days. Students reviewed the material covered on Day 8, and on Day 9, students took a post-test. The lesson plans for the instructional days included a total of eight learning activities, three mini lessons (5-10 minutes of direct instruction), several small group and whole class discussions, and five homework assignments. For descriptions of the activities, see Section 3, and for complete lesson plans, see Appendix B.

Coding

Using a grading rubric, each item of the pre- and post-test was graded on a scale of zero to three. In general, a zero indicated no response or a nonsense response, a one indicated an incorrect response, a two indicated a partially correct response, and a three indicated a correct response. This means that a student could receive a total of 51 points on the pre- or post-test. The conceptual score had a maximum of 21 points, and the procedural score had a maximum of 30 points.

SECTION 3: SWITCHING VIEWPOINTS WHILE LEARNING MATHEMATICS THROUGH DIRECT EMBODIMENT ACTIVITIES

Introduction

Imagine this scenario. A high school mathematics teacher writes the equation, "$32 = -5x^2 + 19x + 60$" on the board and says, "Today we're going to find out what it feels like to be the letter *x*."

It is not often that students get to experience mathematics by becoming a part of it. Most of the time we think of mathematics as so abstract that it is difficult to even imagine ways that it could be experienced—especially at the upper levels. How is it possible to *be* the variable *x*?

Mathematics education researchers have begun to investigate what instruction might look like that allows the student to "become" the mathematics (Abrahamson, Trninic, & Gutierrez, in press). For example, research has shown that enacting distance-time graphs helped students learn about rate of change (Nemirovsky et al., 1998; Noble et al., 2001; Radford, Demers, Guzman, & Cerulli, 2002). While many have suggested that "embodying" mathematics in this way can help students learn, less research has been conducted on how this learning process is different.

I conjecture two main mechanisms by which acting out mathematics helps students learn. First, students are more likely to experience a first person viewpoint by enacting mathematics. When students become actors in mathematics problems, their perspective on the problem changes. Instead of viewing the problem from the outside, they are now "in the action" so to speak.

Second, the students do not maintain a first person viewpoint. Instead, they are prompted to translate back and forth from the first person viewpoint to a third person viewpoint through class discussions and the structure of the task. This process of translating back and forth allows students to develop a deep understanding about the topic.

In this paper, I describe a study comparing high school geometry students learning through "being" mathematics concepts (the embodied condition) to students who learned through more passive activities (the observer condition). I was particularly interested in the differences in the viewpoints students took as they solved problems. My research questions were:

1. Are students in the embodied condition more likely to use a first person viewpoint than students in the observer condition?
2. Are students in the embodied condition more likely to think about mathematics from multiple viewpoints than students in the observer condition?

Background

Embodied Cognition

My study is based around the theoretical framework of embodied cognition. When setting the stage for embodied cognition, researchers often refer to the theory as an emerging alternative perspective to Cartesian dualism, the philosophy which permeates theorizing about learning, thinking and cognitive development. In Cartesian dualism, the mind and the body are separate and disjoint. The body is a machine controlled by the thoughts and decisions made by the mind. The movie *The Adventures of Baron Munchausen* provides a striking example of the separation in our thinking about the functions of our bodies and heads. In the movie, several characters have separable bodies and heads with quite different contributions to the characters' lives. For example, as the seemingly fully capable and in-control head of the King of the Moon flies away from his unruly and mischievous body, the head exhorts, "I'm free! I'm free at last! The body is dead! The body is dead! Long live the head! It's finished, finito, heh-heh! Bye, body! Ha-ha! I shall prove a head does not need a body to survive! I am omnipotent! Ha ha! Yes... OH! Oh no, I got an itch! Oh, no! Oh no, oh no... AH-CHOOOOO!" I love this example because it immediately shows the absurdity of this separation.

Simply put, embodied cognition proposes that all cognitive activity (e.g., thinking, problem solving, learning, and conceptual development) arises from the interactions of a brain situated in a body and an environment, not just a brain alone. The claims of embodied cognition vary widely; however, Wilson (2002) has outlined six main tenets of the field: cognition is situated, cognition is time pressured, we off-load cognitive work onto the environment, the environment is part of the cognitive system, cognition is for action, and off-line cognition is body-based.

The underlying foundations of embodied cognition research date back to philosophers Merleau-Ponty and Heidegger and psychologists Dewey, Piaget and Vygotsky, and in the last two decades, the body of research on the subject has grown substantially. Important works in cognitive psychology (Barsalou, 1999, 2008; Varela, Thompson, & Rosch, 1993), linguistics (Lakoff & Johnson, 1999; Lakoff & Nuñez, 2000), and neuroscience (Gallese, Fadiga, Fogassi, & Rizzolatti, 1996; Rizzolatti, Fadiga, Gallese, & Fogassi, 1996) have made significant progress working out many of the elements necessary for the theory to have solid explanatory power.

For example, in alternative symbolic systems, Barsalou argues against the dominant amodal (i.e. non-perceptual) model of cognitive thought processes and in favor of a perceptual symbol system. According to this system, concepts that form our thought processes are represented perceptually rather than with abstract symbols. Simulations or reenactments of our perceptual experiences underlie all knowledge representation and thought processes. In this model, embodiment is central to cognition.

In the field of cognitive linguistics, researchers use predominantly linguistic evidence to propose a theory of cognition based on conceptual metaphors that are grounded in bodily experiences (Lakoff & Johnson, 1980; Lakoff & Nuñez, 1997, 2000). These metaphors structure our thoughts and actions and help us understand the world around us. Lakoff and Nuñez (2000) even argue that we make sense of abstract ideas and mathematics concepts through layers of conceptual metaphor that relate back to concrete experiences. For example, they explain that we understand arithmetic through the metaphor "Arithmetic is Motion Along a Path." In this metaphor, the origin of the path is zero, and points along the path are numbers. Moving from a point, X, on a path towards the origin to a point, Y, is the subtraction of Y from X.

Neuroscience research also provides support for an embodied model of cognition. The discovery of mirror neurons, which fire both when an action is observed and when it is performed, supports the idea that perception and action are not separate processes but are intertwined (Gallese et al., 1996; Rizzolatti et al., 1996). In other words, when we observe someone else perform an action, we make meaning of that action, in a sense, by simulating it ourselves. The same neural substrates are activated when we observe, for example, someone pick up a pencil as when we pick up a pencil ourselves. This provides further evidence that our cognitive processes are grounded in the actions of our body.

Thomas & Lleras (2009) found that when study participants were directed to make physical movements that related to a problem solution, they were more likely to solve the problem than when they made unrelated movements—even when they were not aware that their actions were related to the problem solution. Furthermore, Goldin-Meadow et al. (2009) found that teaching children mathematically correct gestures while learning a new strategy for solving addition problems was more effective than just learning the strategy by itself. These examples offer exciting opportunities for teaching and learning, and I was interested in testing whether and, more specifically, how explicitly relating students' actions with content could improve learning.

Direct Embodiment

An issue that arises in this idea of "testing" embodied learning is that I do not posit a learning environment that is not embodied. Following the theoretical claims of embodied cognition, all cognitive processes including problem solving, remembering, imagining, and introspection are embodied (Barsalou, 2008; Gibbs, 2005; Glenberg, 1997). However, I am interested in comparing activities that provide greater and lesser affordances for engaging the benefits embodiment could provide while learning novel content in geometry. In other words, I want to maximize the chance of tapping into the affordances of embodiment for learning something new.

To do this, I chose to compare learning through direct embodiment (i.e. physically enacting or “being” a concept, see Fadjo et al., 2009) with more passive instructional activities. There is evidence that relating physical experiences with abstract concepts can help students develop their conceptual understanding of those concepts (Abrahamson & Howison, 2010; Gerofsky, in press; Han, Black, & Hallman, 2009).

Experts regularly employ similar strategies. Physicists transport themselves into a problem by talking and moving their bodies when they interpret graphs (Ochs, Jacoby, & Gonzales, 1994). Barbara McClintock, a Nobel Prize winner for discovering genetic transposition in corn, credited her achievements to her "feeling for the organism" and envisioned herself as the organisms she was examining (Keller, 1983). Theoretical astrophysicist, Jacob Shaham, imagined his equations were scripts of plays that he could read and enact, and the parts of the equations were personified with motives that he took on as he played each role (Root-Bernstein & Root-Bernstein, 2003). Even Albert Einstein professed that at age 16 he imagined himself chasing a beam of light. He said taking such a perspective was instrumental in shaping his thinking about special relativity (Einstein, 1951).

In addition, intriguing empirical work has been done investigating how direct embodiment could help students learn in a variety of subject areas. In mathematics, Howison, Trinic, Reinholz, & Abrahamson (2011) found that students developed ideas about proportional equivalence through manipulating Wii remotes with their hands. When the students raised the Wii remotes so that their heights reached a certain ratio, a screen in front of them turned from red to green. Wright (2001) describes a case of a student acting out motion trips along a path. Physically acting out parts of the problem lead her to rethink her initial incorrect responses and prompted new insights about rate of change.

In computer science, Petrick, Berland, and Martin (2011) describe how high school students directly embody virtual robots to help them develop program code, and Fadjo et al. (2009) show how directly embodying program scripts lead to the implementation of more complex conditional sequences for third- and fourth-graders learning to program.

In science, Huang, Black, and Vea (2012) compared two groups of students learning about conservation of energy. Both groups learned by watching a flash simulation and using a 3-D force feedback joystick. One condition experienced force feedback from the joystick, while the other did not. They found that the group that physically experienced force feedback had greater learning gains on a pre- and post-test than the group that did not get the physical feedback. Birchfield & Megowan-Romanowicz (2009) examined how high school earth science students collaborate in an embodied learning environment called SMALLab. They compared the number of student and teacher exchanges in the regular classroom to the SMALLab environment and found a greater percentage of student-to-student exchanges in SMALLab than in the classroom.

These studies demonstrate that students can indeed learn through directly embodying concepts they are studying. However, most of these studies take place outside of the classroom in controlled environments. In addition, several of the studies lack a control group or have very few participants. I designed a more robust study to find out how direct embodiment might play out in a school setting and how it would compare to more passive forms of instruction.

In my study, high school geometry students (ages 14-19) participated in a two-week unit on similarity under two different conditions (embodied and observer). Both conditions had similarly structured activities with identical content. However, the students in the embodied condition learned through direct embodiment instructional activities. This differed from the students in the observer condition who did not physically act out mathematics. Rather than having opportunities to directly embody the mathematics, they observed abstract simulations of mathematics concepts or drew pictures of the concepts.

As described in Section 2, I found that students in the embodied condition had significantly greater overall learning gains. I also found that students in the embodied condition had significantly greater gains on the conceptual items on the test, while there were no significant differences between the conditions on the procedural items.

In this paper, I investigate differences in the learning process that might account for the differences I found in learning gains. Specifically, I look at the perspectives or viewpoints on mathematics and activity that students took during the unit, and I examine the types of details students remembered about the activities in which they participated.

Switching Viewpoints

Students can adopt different viewpoints when solving mathematics problems (Srisurichan et al., under review). By viewpoint, I mean “locus of consciousness for a model of the world” (Parrill, 2009, p. 272). For example, if a student were trying to estimate an angle measurement, a common approach would be to adopt an external viewpoint on the problem. The student might place the corner of a piece of paper at the vertex of the angle, align one side of the angle with one edge of the paper, and look to see if the other side of the angle opens out beyond the other edge of the paper (See Figure 3.1).

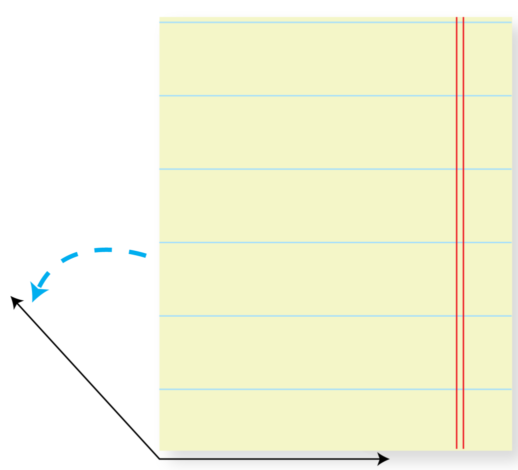


Figure 3.1. Using a piece of paper to estimate an angle measurement.

In this case, the student's locus of conscious remains separate from the mathematical activity, and the student is comparing the measure of the angle to the corner of the paper.

Alternatively, the student could take an internal viewpoint to determine the angle measurement. She could imagine that she is the vertex of the angle. If she were to reach her right arm straight out to her side and extend her left arm directly in front of her, then she knows her arms would form a 90 degree angle. To estimate the angle measurement, she might ask herself if she would need to open her left arm farther or close it in towards the other arm in order to form the angle shown (See Figure 3.2). The student might even physically act out the movements with her arms to help her envision the scenario.

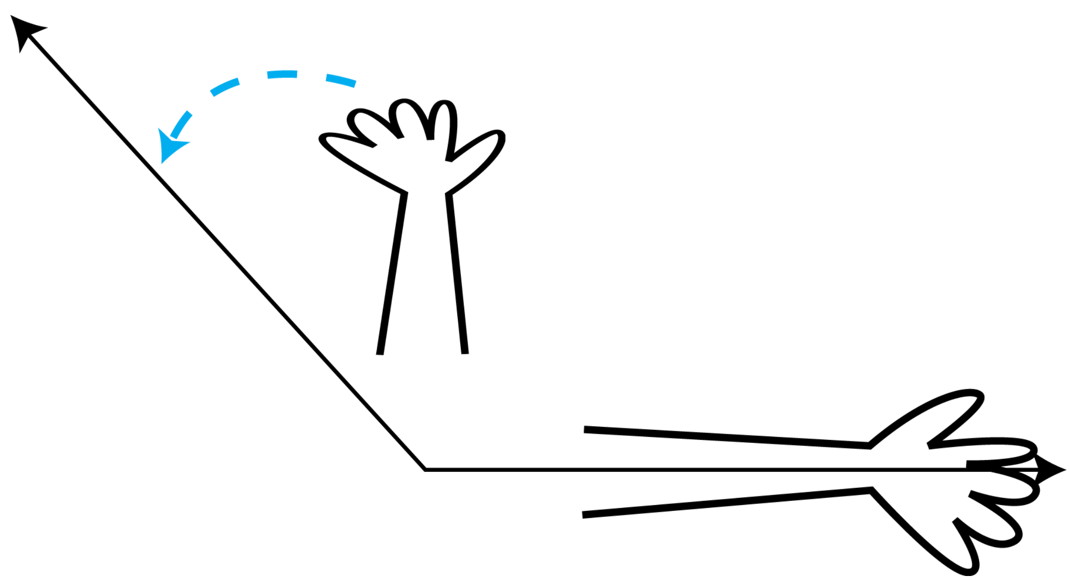


Figure 3.2. Using a first person viewpoint to estimate an angle measurement.

Here the student's locus of consciousness has shifted "inside" the problem. The student has become part of the problem, as her arms *are* the sides of the angle.

Hall (1996) calls the external view a third person or observer perspective and the internal view a first person or participant perspective. These correspond closely with the character and observer viewpoints described in Srisurichan et al. (under review), which are derived from McNeill's (1992) work on gesture viewpoints.

Secondary mathematics instruction almost exclusively promotes a third person viewpoint (Srisurichan et al., under review). Traditional class activities such as interpreting diagrams and graphs, manipulating equations, and solving problems do not afford translation into the first person. As in the example above describing the student measuring an angle, it is possible that students can mentally transition into a first person viewpoint on their own; however, these kinds of traditional activities typically lack any direction that would prompt students to do so.

In contrast, physically representing part of a problem has a much greater affordance of translation to a first person viewpoint. This viewpoint plays an important role in learning (Gerofsky, 2010). Adopting a first person viewpoint while studying mathematics may illuminate aspects of the content that were hidden from a third person viewpoint. For example, Papert (1980) describes a scenario of a student using the computer program Logo to create the image of a circle on the screen. The student writes a program to control a virtual "turtle" that traces its movements on the screen. To determine how to program the turtle, the student first steps away from the computer. Then she translates to a first person viewpoint, imagines she is the turtle drawing the circle, and makes the steps she wants the turtle to make. By enacting the turtle's path, the student discovers new ideas for writing her code. She returns to the computer and enters in the code for the circle.

Viewing a problem from the inside can help students understand it more fully (Wright, 2001). Srisurichan et al. (under review) found evidence that adopting a first person viewpoint may scaffold students to a better understanding of mathematical proof. In contrast, Goldin-Meadow and Beilock (2010) suggest that the third person viewpoint emphasizes more abstract qualities, which could promote generalization and transfer.

Both viewpoints have different affordances for students; however, there is little research on learning through multiple viewpoints, particularly in the area of mathematics (Srisurichan et al., under review). Goldin-Meadow and Beilock (2010) suggest that teaching children to gesture from a first person viewpoint followed by producing gestures from an third person viewpoint might help students move from concrete to more abstract representations.

I agree that instruction which incorporates both viewpoints is important; however, I hypothesize that alternating back and forth between them, rather than starting with one viewpoint and ending with another, will best support learning. While it is possible (and common) to learn and remain in one viewpoint, switching to another viewpoint can emphasize different parts of a problem. When a student switches back and forth between viewpoints she is constantly integrating the information from one viewpoint into the other developing a complete picture. My research in the area of computer science describes a case of a student programming a robot to play soccer (Petrick et al., 2011). The student starts in a third person viewpoint thinking about the program as a whole. When debugging her program, the student switches to a first person viewpoint, directly embodies the robot, acts out its movements, and through her actions, decides exactly how her robot should move. When the student switches back to a third person viewpoint, she applies the new insight she gained from the first person viewpoint. Switching back and forth between viewpoints helped this student and others develop better programs.

This interchange between "diving-in" and "stepping-out," as described by Ackermann (1996), is important for developing conceptual understanding. Ackermann (2004) describes how we need time diving-in and immersing ourselves, examining problems from a new perspective. Then, to fully digest what we have taken in, we must step back and observe at length. This switching of perspectives is a natural part of human development (Kegan, 1982).

Different viewpoints have different affordances, and being able to switch back and forth between viewpoints and translate information from one viewpoint to another could be important in helping students to make connections within a concept. Indeed, when Ochs, Gonzales, & Jacoby (1996) analyzed the speech and gestures of physicists during discussions at lab meetings, they found that the physicists naturally transitioned between both first person and third person viewpoints when trying understand each other. The same occurrence has been found in students discussing science (DeLiema & Enyedy, in press). In similar research, Cassell & McNeill (1991) describe how study participants alternated seamlessly and purposefully between viewpoints while recounting a story.

More important events were told from a first person viewpoint, while less important details were described from an third person viewpoint.

In my study, I predicted that students in the embodied condition would be more likely to adopt a first person viewpoint while learning than students in the observer condition because the direct embodiment activities would place them right in the middle of the action. I also thought that the mathematical discussions these students had with their classmates and the teacher would afford translation to a third person perspective. Given prompts for students in the embodied condition to translate to both first person and third person viewpoints, I predicted that they would switch back and forth more often than students in the observer condition, explaining their greater conceptual learning.

In contrast, I thought students in the observer condition would be more likely to maintain a third person viewpoint since they were never instructed to physically enact the mathematics, and they would be less likely to switch viewpoints.

Geometry

Given its spatial nature, geometry is a natural domain in which to examine direct embodiment (Srisurichan et al., under review). I chose to focus on one important topic in geometry: similarity. In mathematics, two figures are similar if they have corresponding angles that are congruent and corresponding sides in the same ratio. I focused my study to this area for two main reasons: first, the teachers participating in this study identified similarity as a difficult topic for their students in the past, and second, understanding similarity means having a strong proportional reasoning skills—an area for which embodied actions can support learners (Abrahamson & Howison, 2010).

Proportional reasoning is central to middle school and high school mathematics (Lesh, Post, & Behr, 1998), yet it is something many adults even struggle with (Lamon, 2005). There has been extensive research examining how students develop proportional reasoning skills (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Inhelder & Piaget, 1958; Lamon, 1993; Streefland, 1985; Tourniaire & Pulos, 1985). One of the most common errors students make when solving proportion problems is to use an additive or fixed difference strategy rather than a multiplicative one (Kaput & West, 1994; Lamon, 1993; Tourniaire & Pulos, 1985; Vergnaud, 1982). For example, a student might argue that 2/3 is equivalent to 5/6 because if three is added to the numerator and the denominator of 2/3, the result is 5/6 (See Figure 3.3).

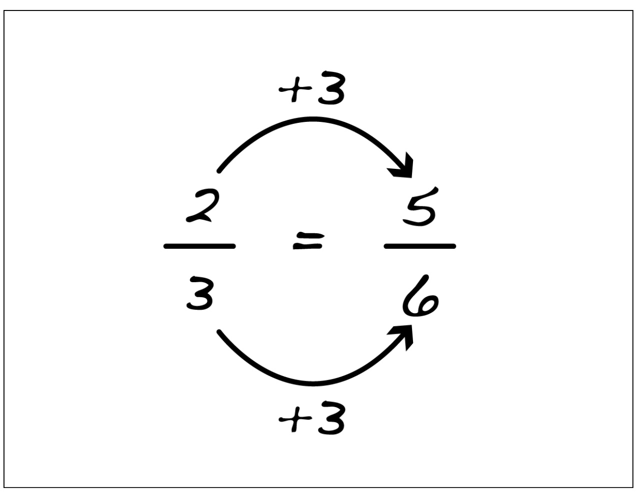


Figure 3.3. Example of additive/fixed difference reasoning.

Some proportional reasoning problem types are more difficult for students than others. Stretcher and shrinker problems are particularly difficult (Lamon, 1993). These types of problems compare two quantities that are related but are not part of the same whole, and they involve growth or similarity transformations of figures (Ben-Chaim et al., 1998). Students tend to use lower level strategies, particularly additive strategies, when working these types of problems—even when they have advanced to applying higher level reasoning to easier problems (Lamon, 1993). Building my curriculum around geometric similarity allows me to focus particularly on stretcher/shrinker problems.

Experts suggest that instruction on proportionality should include a variety of problem types and structures (Tourniaire, 1985), give students opportunities to construct their own problem solving strategies (Ben-Chaim et al., 1998), and build upon students' intuitive ideas to more abstract concepts (Streefland, 1985).

While utilizing a range of problem types and encouraging students to develop their own approaches and ways of thinking about problems can be incorporated in many instructional contexts, direct embodiment activities have the potential to especially address and incorporate students' intuitive ideas.

Direct embodiment offers students the opportunity to repurpose their physical experiences in the world as resources for mathematical thinking in the classroom. Nuñez, Edwards, and Mateo (1999) call for mathematics instruction that takes into account students' embodied knowledge. For example, the Dutch have restructured their geometry curriculum over the past four decades with the intention of drawing upon students' intuitive understanding of their environments (Gravemeijer, 1998). Furthermore, Srisurichan et al. (under review) describe how directly embodying parts of a triangle can lead to stronger geometric proofs.

In addition, embodied representations of geometric figures formed by the students' bodies create visual representations to use for solving problems. The importance of visual representations in mathematics, especially in geometry, cannot be understated (Arcavi, 2003), and there is an added benefit towards learning when students produce these models themselves (Streefland, 1985).

Methods

Participants

Students from 14 geometry classes in the same urban high school were invited to participate in this study. This paper will report on data from 148 of these students who consented to participate and completed the survey at the end of the study. The students did not receive compensation for participation. They ranged in age from 14-19 years old (M = 16.3, SD = 1.08). 55 were male and 93 were female.

The school's four geometry teachers also participated in the study. The teachers had an average of 10 years of teaching experience (SD = 8.8), and I conducted a 13-hour professional development series with the teachers as part of their participation in the study. I randomly assigned half of each teacher's classes to the embodied condition and other half to the observer condition. Since two teachers taught an odd number of classes, one teacher taught an extra observer class, and one teacher taught an extra embodied class. Having each teacher teach classes in both conditions minimized any potential teacher effects.

Materials

The two-week similarity unit included 8 learning activities, which comprised the bulk of the instruction time, 3 short teacher-directed lessons, several small group and whole group discussions, and 5 homework assignments. The instruction for the two conditions was identical with the exception of one key difference: the 8 learning activities for the embodied condition had students physically represent and act out mathematics concepts, while the eight activities for the observer condition had students draw or observe abstract representations of the same concepts. Aside from this difference, the structure of each pair of activities was as similar as possible for the two conditions. Though they modeled the mathematics in different ways, the students in both conditions completed the same mathematical tasks, worked in the same size groups, and their teachers gave them the same prompts and discussion questions.

Both the mathematics content and quality of instruction was determined equivalent by independent mathematics and education experts, respectively. Here I give short descriptions of each activity. For further descriptions and additional details about the lesson plans, see Appendix B.

Activity 1 – Make the Screen Green. The *Make the Screen Green* activity was based off of Abrahamson's Mathematical Imagery Trainer (MIT) (Abrahamson & Howison, 2010; Howison et al., 2011). The MIT is designed to give students embodied experiences of proportional relationships and to specifically target students' incorrect tendencies towards using additive strategies (Abrahamson & Howison, 2010).

Embodied version. In the embodied condition, students used Wii remotes to control the heights of blocks on a screen (See Figure 3.4).

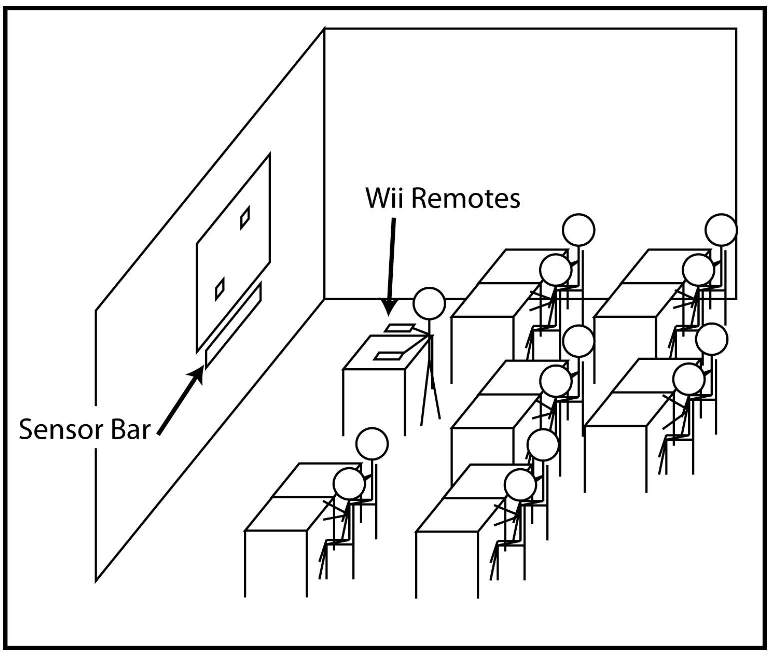


Figure 3.4. Classroom set-up for embodied version of Make the Screen Green. Students took turns using the Wii Remotes to control the heights of the blocks on the screen.

At the beginning of the activity, the screen was red, but when the ratio of the heights of the blocks was 2:1, the screen turned from red to green (See Figure 3.5). The students had to determine this hidden rule by taking turns moving the remotes.

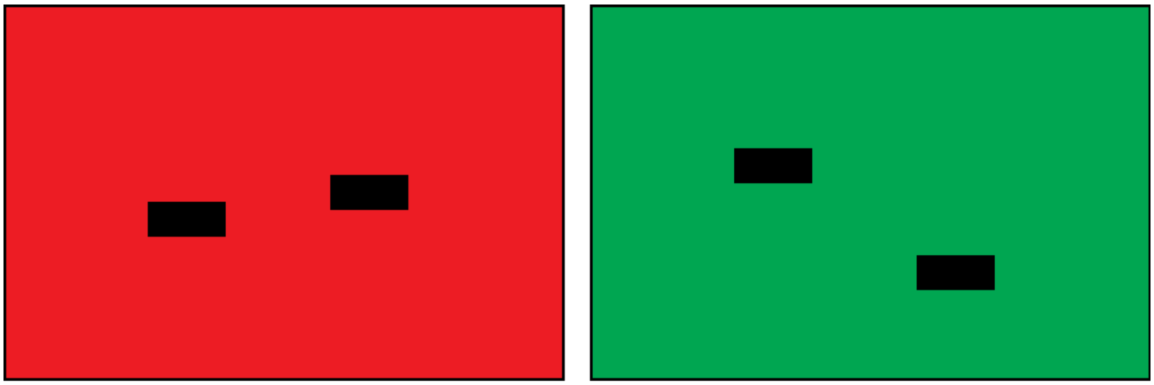


Figure 3.5. Make the Screen Green Activity screen images. When the heights of the blocks were not in a 2:1 ratio (left), the screen turned red. When they were (right), the screen turned green.

Every few minutes the teacher instructed the entire class to use their hands to model the configuration of the blocks that they thought would make the screen turn green. Students discussed their ideas with a partner and shared with the class. About two thirds of the way through the activity, the teacher added an overlay to the screen with a vertical axis labeled with numbers and horizontal lines going across the screen (See Figure 3.6). This prompted students to generalize their ideas about the hidden rule using mathematical language. Throughout the activity, the teacher never confirmed, nor denied any of the students' hypotheses about the hidden rule. Instead, the teacher prompted students to collect more data to test their ideas.

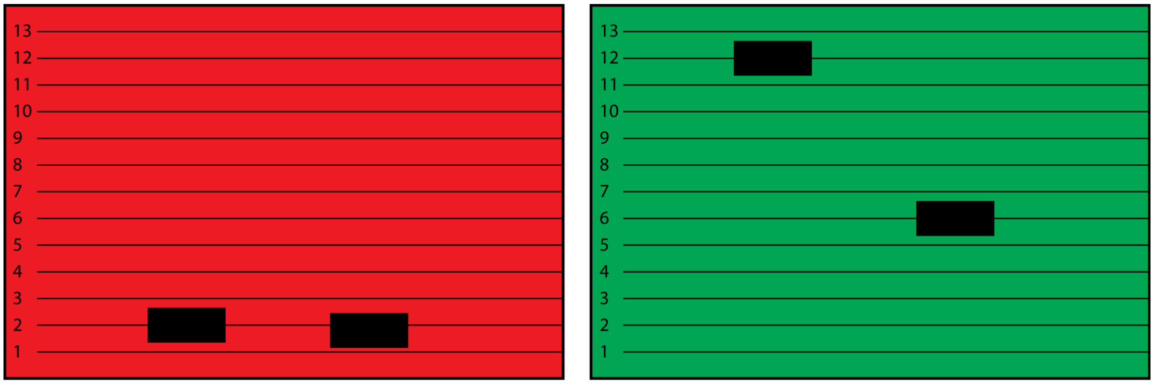


Figure 3.6. Examples of the screen with overlay.

Observer version. The students in the observer condition did not use the Wii remotes to control the blocks on the screen. Instead the students watched a video showing a typical progression of block movements that students in the embodied condition made as they became more and more familiar with the activity (See Figure 3.7). Random movements of the blocks became more and more refined over time. In this way, the observer condition saw the same thing as the students in the embodied condition; however, they were not embodying the blocks' movements with their hands.

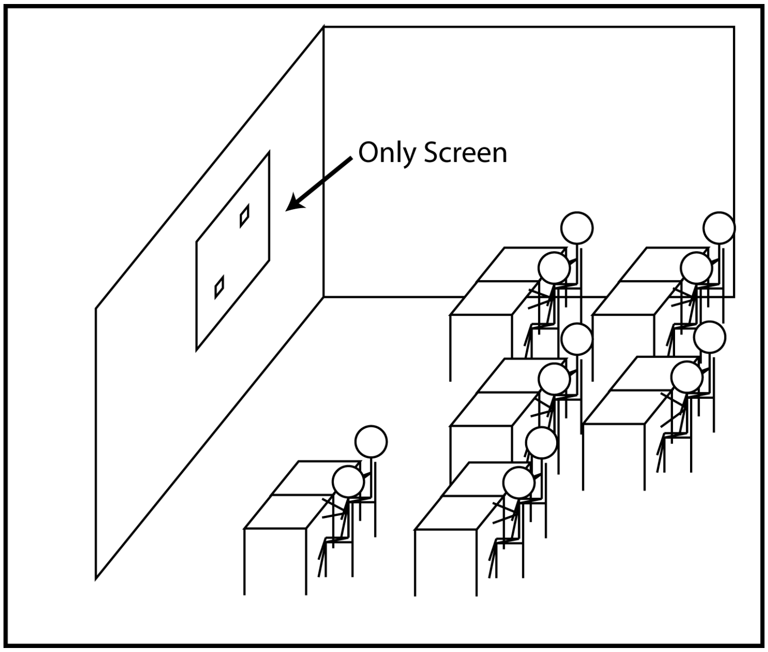


Figure 3.7. Classroom set-up for observer version of Make the Screen Green.

Everything else about the activity was the same. The students discussed their ideas about the hidden rule with a partner. They could ask the teacher to pause, rewind, or fast-forward the video to test their hypotheses, and the teacher added an overlay with numbers on the screen about two thirds of the way through the activity. Students in each of the classes in both conditions brought forth the idea of a 2 to 1 relationship between the heights of the blocks.

Activity 2 – Ratio Red Light Green Light/Ratio Race. I included a second activity dealing with proportional relationships to allow students further time to struggle with the concept (Ben-Chaim et al., 1998) and to present students an opportunity to work with proportions in another context (Tourniaire & Pulos, 1985).

Embodied version. This activity took place in an open room with desks and chairs pushed to the side. Students lined up with a partner on one side of the room behind a starting line. The teacher gave the class a target ratio such as 3:4. When the teacher said, “Green light,” the students walked forward, but when the teacher said, “Red light,” the students stopped where they were. The goal of the students was to coordinate their steps with their partner’s so that their distances from the starting line maintained the target ratio. For example, if the target ratio was 3:4, this meant the ratio of the distance of student A to the starting line to the distance of student B to the starting line needed to always be 3:4 as they moved forward. When the students stopped at a red light, each pair of students would look at their positions and estimate how close they were to the target ratio (See Figure 3.8).

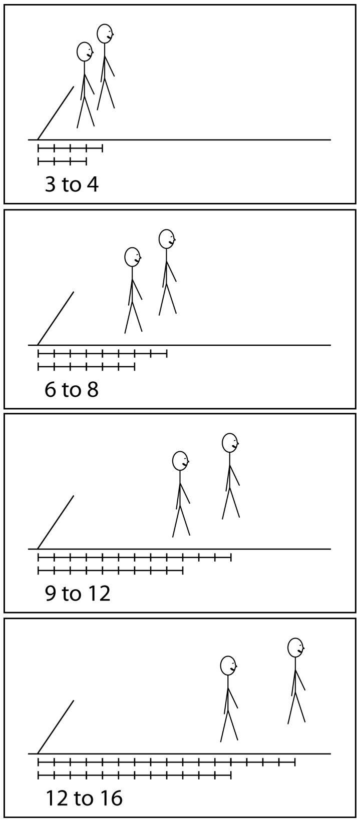


Figure 3.8. Two students walking forward in the Ratio Red Light Green Light/Ratio Race activity. In this example, the ratio of the students' distances to the starting line is always 3:4.

Observer version. The students worked with a partner and watched two dots on a screen that started behind a line and moved to the right (See Figure 3.9). The teacher gave the class a target ratio, and the teacher would start and stop the dots in a similar manner as the embodied condition. The goal of the students was to determine whether or not the distances from the dots to the starting line maintained the target ratio.

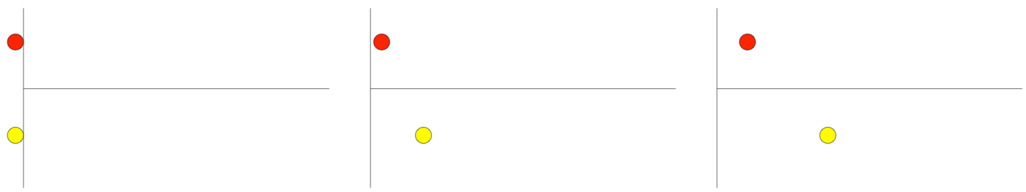


Figure 3.9. Dots moving in the Ratio Red Light Green Light/Ratio Race activity.

Activity 3 – Ratio Challenges. Rather than emphasize standard algorithms for solving proportions, this activity was structured to give students room to create their own problem solving strategies for proportional reasoning problems (Ben-Chaim et al., 1998) and to move students to more generalized strategies (Streefland, 1985).

The students worked in groups of three to solve two challenges, both part-part-whole proportion problems, set in the context of the Ratio Red Light Green Light/Ratio Race activity (i.e. The scenario for the embodied condition involved a pair of students walking, and the scenario for the observer condition involved two dots moving across a screen). The first part of each challenge had a simple numerical structure that could be easily modeled. The numerical structure for the second problem was more complex. I chose these two problems to scaffold students to more complex proportional strategies. Students often solve part-part-whole problems using direct modeling and building up strategies (Lamon, 1993). The students were asked to directly model the first part of each challenge. Students in the embodied condition directly embodied the problem, while students in the observer condition drew pictures. The second part of each challenge included much larger numbers that would be difficult to model or act out, which lead to students using more sophisticated strategies to solve them.

Activity 4 – Who Went Further?/Which One Went Further? To help students distinguish between additive and multiplicative situations (Kurtz & Karplus, 1979), I developed the Who (or Which one) Went Further? task based on one of the Probe Tasks from Royal Melbourne University's Supporting Indigenous Student Achievement in Numeracy Project (Commonwealth of Australia, 2005).

Embodied version. Students worked with a partner to act out a problem described as follows: On move 1, student A takes 2 steps, and student B takes 4 steps. On move 2, student A takes 4 steps, and student B takes 4 steps. Who went further on the second move? The majority of students initially answered that both students went the same amount on the second move, but the teacher encouraged students to see if they could think about the problem in a different way and prompted them to think about the second move in terms of or compared to the first move. The teacher facilitated a class discussion on the difference between thinking of the problem in terms of an additive or multiplicative situation.

Observer version. The students completed an activity that was identical except that instead of acting out the problem, the students observed abstract dots moving on a screen.

Activity 5 – Growing & Shrinking Triangles. The purpose of this activity was for students to develop and test a rule for triangles of the same shape, which served as an introduction to the definition of similarity.

Embodied version. Students worked in groups of three using a "measuring tape triangle" composed of three measuring tapes attached end to end. The measuring tapes could be extended or retracted to form triangles of different sizes. Each student represented a vertex of the triangle and held onto the base of one of the measuring tapes (See Figure 3.10). The students were instructed to make a triangle, enlarge it until each side was twice its original size, and observe what any changes to the angles. The students followed other similar exercises creating triangles of different sizes to gather data about the angles and sides in different situations. Then the students synthesized their observations and created a rule for triangles of the same shape.

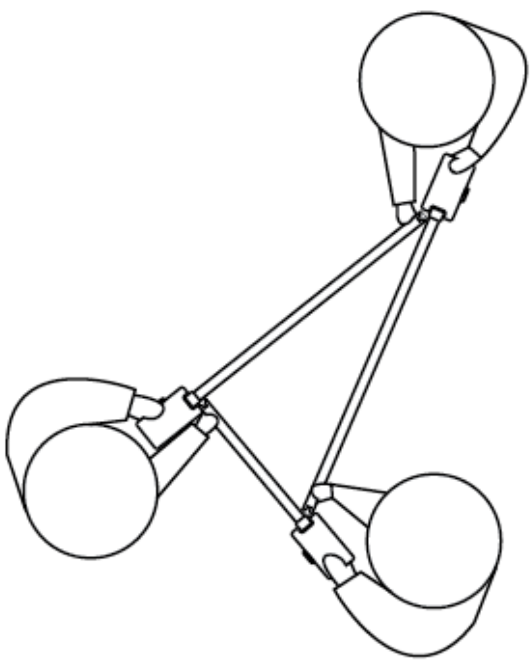


Figure 3.10. Measuring tape triangle. Three students use the measuring tape triangle to form a triangle with each student representing a vertex of the triangle.

Observer version. Students worked in groups of three and used rulers and protractors to create triangles. The students followed the same set of exercises as students in the embodied condition, making triangles of different sizes and recording observations about the side lengths and angle measures. They too developed a rule for triangles of the same shape that the teacher then connected to the definition of similarity.

Activity 6 – Musical Triangles/Similar Triangles. This activity was based off of an activity used by a previous teacher at the school that was designed to help their students read mathematical notation and identify corresponding angles and sides. The goal of this activity was to familiarize students with the triangle similarity postulates (Angle-Angle, Side-Angle-Side, Side-Side-Side). This activity followed a class discussion about the information necessary to prove two triangles were similar and an introduction to the similarity postulates.

Embodied version. This activity took place in an open room with the desks and chairs pushed to the side. Each group of three students had a large, poster board triangle on the ground near them. The vertices of this triangle were labeled X, Y, and Z. Music would play, and the students would walk around the triangle. When the music stopped students sat on either a side or angle of the triangle with their arms stretched out in the form of the part of the triangle they were on. While the students were walking around the triangle, they looked at the information presented on the screen to determine whether or not it was possible to prove that the two triangles were similar (See Figure 3.11).

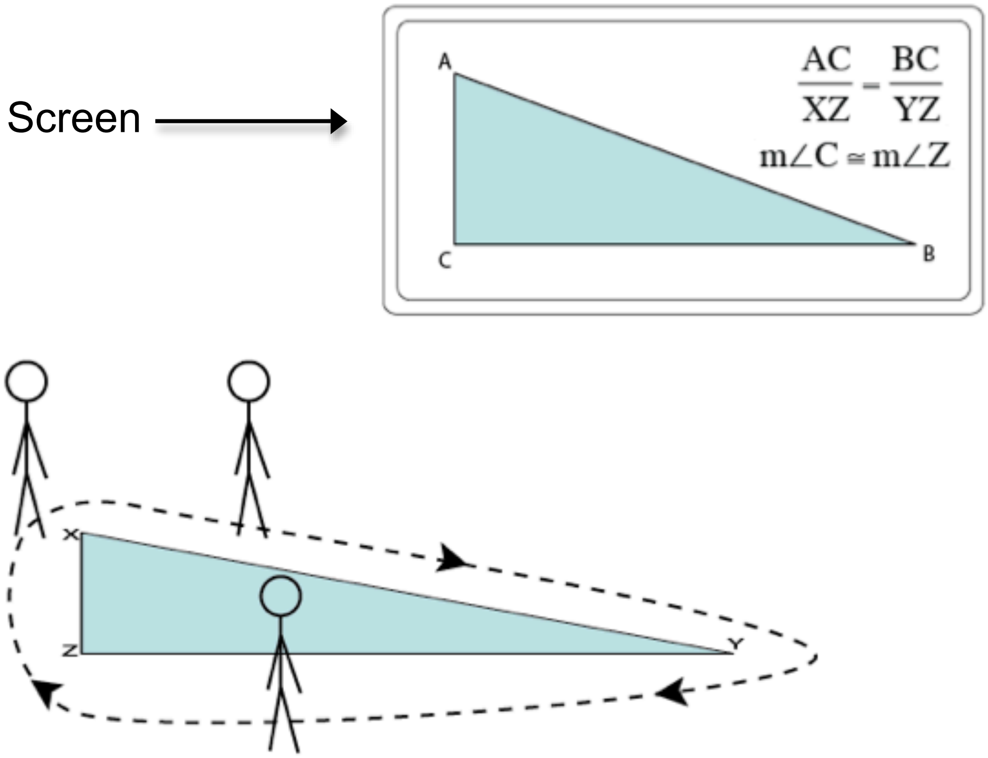


Figure 3.11. Students walking around a triangle on the floor. They are trying to determine whether triangle XYZ (on the floor) is similar to triangle ABC (on the screen) based on the information given.

In the example in Figure 3.11, the two triangles are similar based on the Side-Angle-Side triangle similarity postulate. When the music stops, the students must stand on a side of triangle XYZ that is proportional to a side of triangle ABC or an angle of triangle XYZ that is congruent to triangle ABC. While it is true that since the two triangles are similar, all pairs of corresponding sides are proportional and all pairs of corresponding angles are congruent, the students were instructed to choose the angles and/or sides that helped them determine the two triangles were similar. In this example, that would be angle Z and sides XZ and YZ (See Figure 3.12).

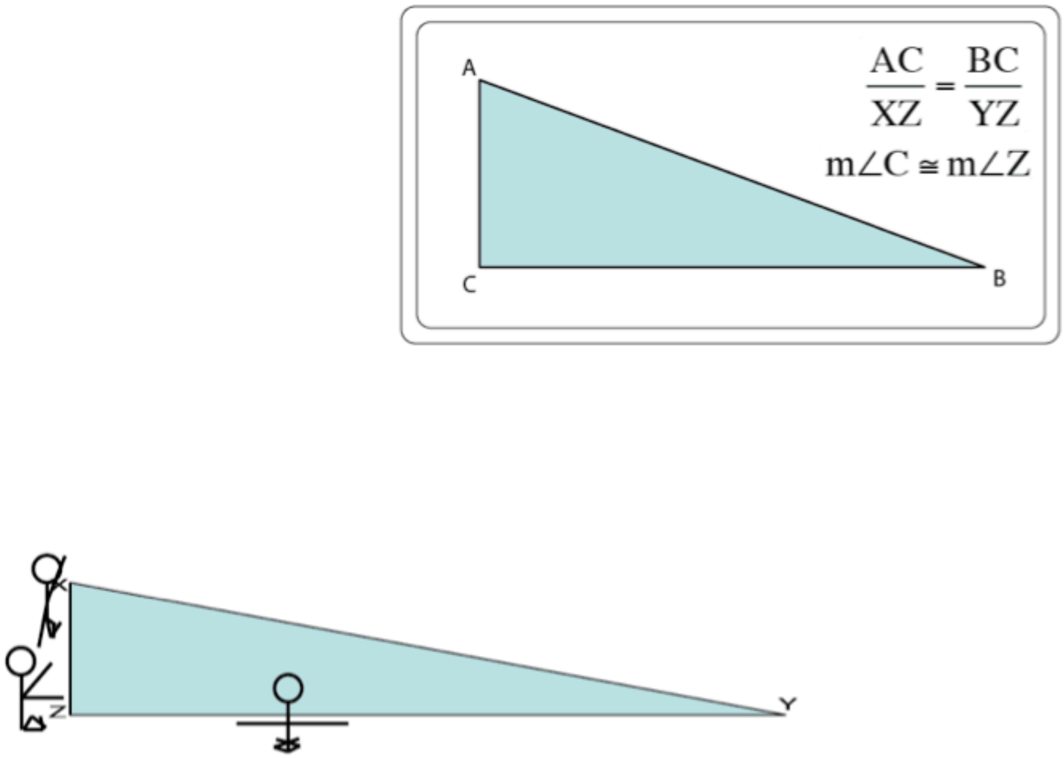


Figure 3.12. Students sitting on a triangle on the floor. They are embodying the parts of triangle XYZ that can be used to prove it is similar to triangle ABC by the Side-Angle-Side similarity postulate.

After the students have found their places on the triangle, the teacher asked different groups of students to explain where they had chosen to sit and why. For certain triangles on the board, there were multiple postulates that could be used to prove the triangles were similar (and sometimes the triangles were not similar). The students had to justify their choices to the class.

Observer version. Instead of having triangle XYZ on the floor, the observer condition had it on their handout. They looked at the screen and saw the same information as the students in the embodied version, but instead of physically creating sides and angles, they used colored pencils to mark the sides and/or angles they used to show the two triangles were similar (See Figure 3.13).

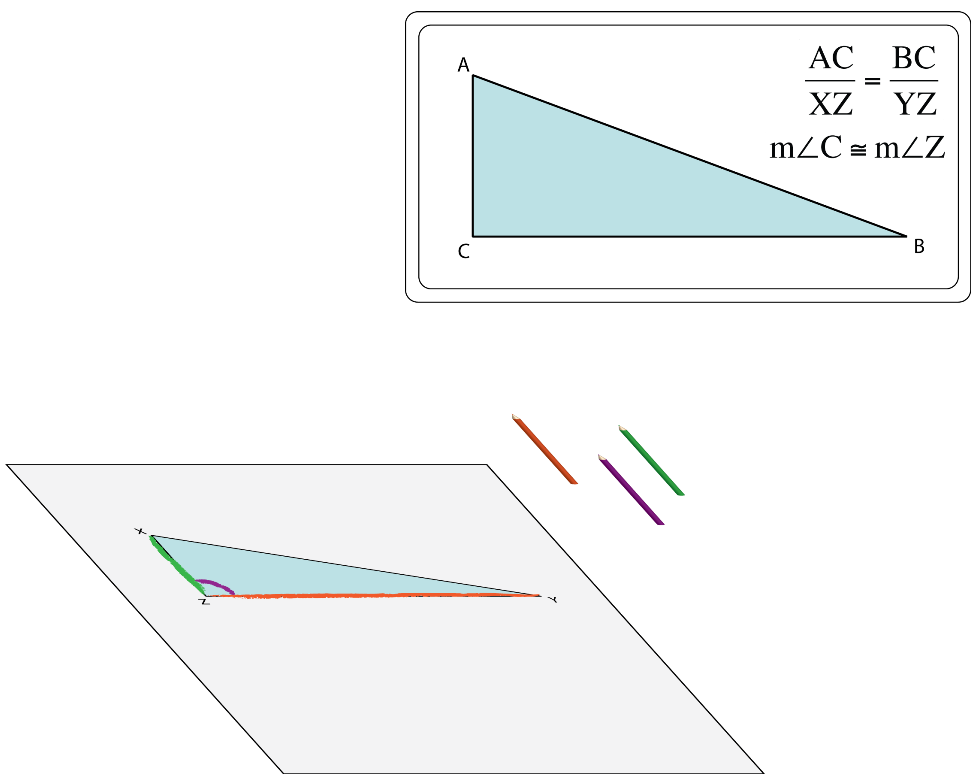


Figure 3.13. Set-up for the observer version of the Similar Triangles activity. The students used colored pencils to indicate the parts of triangle XYZ that they used to prove it was similar to triangle ABC by the Side-Angle-Side similarity postulate.

Activity 7 – Indirect Measurement. This activity was adapted from Streefland's (1985) modeling activity for measuring an inaccessible height. Students used proportional relationships to find distances that could not be measured directly.

Embodied version. In the embodied condition, groups of students developed a strategy to measure the height of a tall object using only their bodies, a mirror, and a measuring tape. Each student first placed a mirror in between themselves and the object. The student then moved so that she could see the top of the object in the mirror. This formed two similar triangles in which the height of the student's body (from the ground to her eyes) was proportional to the height of the tall object (See Figure 3.14). The student could use the measurements to calculate the height of the object.

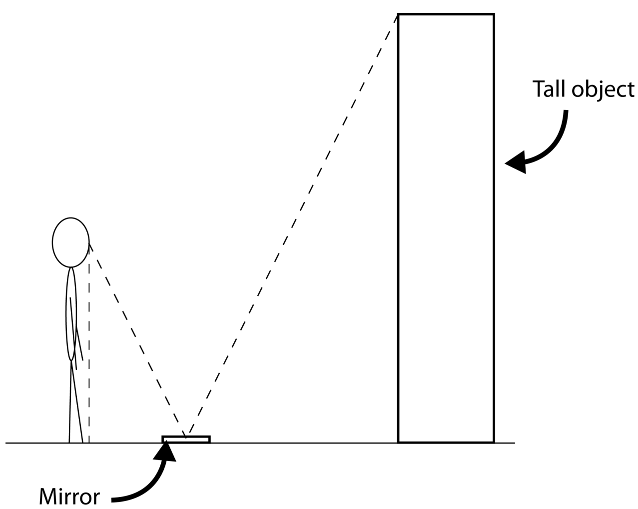


Figure 3.14. Embodied version of indirect measurement activity. A student directly embodies a side of a triangle in order to measure the height of a tall object.

Observer version. The students in the observer condition participated in a similar activity; however, instead of acting out the problem, they were instructed to use a sketch on paper. Students worked in groups to develop a strategy to measure a tall object using a laser mounted on a tripod, a mirror, and a measuring tape (See Figure 3.15). Once students had completed a sketch and developed a strategy to find the height of the tall object, the teacher gave them any measurements that they requested for use in their calculations.

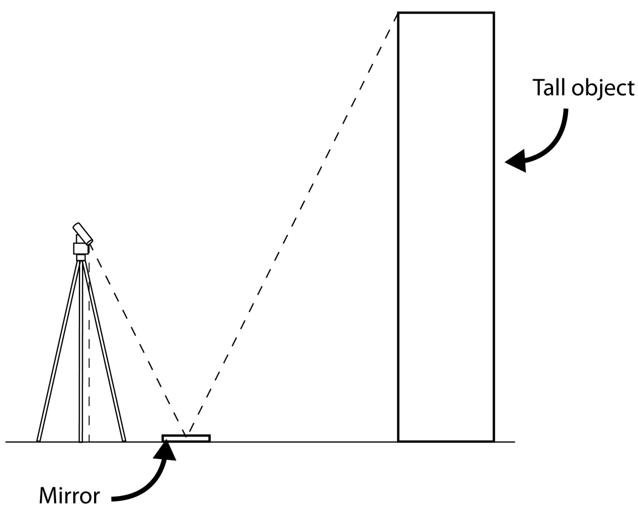


Figure 3.15. Observer version of indirect measurement activity. Rather than directly embodying the problem, students in the observer condition used a laser reflecting off of a mirror to measure the height of a tall object.

Activity 8 – What If? This activity was designed as an extension of the Indirect Measurement activity in which students made predictions based on proportional reasoning. This was the last activity in the unit, and in many classes, it was rushed because teachers were running out of time.

Embodied version. The teacher displayed a diagram on the screen (See, for example, Figure 3.16), and asked to predict how they would have to move to see the top of the object in the mirror if they were the person in the diagram. The students could act out the scenario using a mirror if they wanted to, but due to time constraints, after the first problem, the teacher often modeled the rest. The class went through several similar scenarios, and students discussed each with a partner.

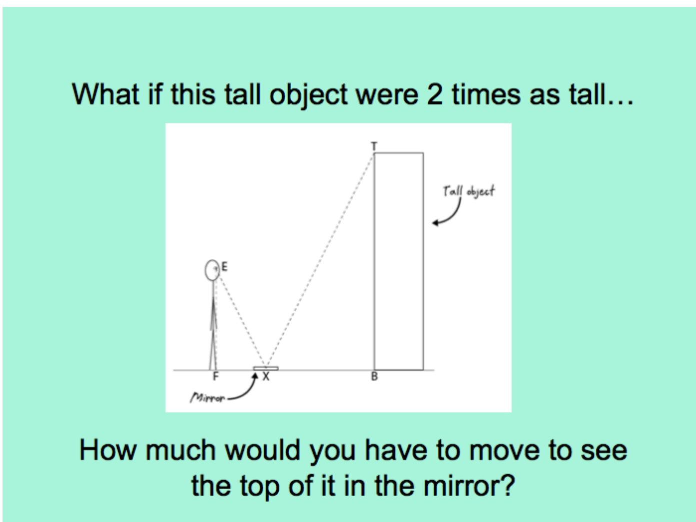


Figure 3.16. Example slide from embodied version of the What If? Activity.

Observer version. In the same way as the Indirect Measurement activity, the main difference between the two conditions for the What If? activity was that rather than a person looking at a reflection in the mirror, the scenario for the observer condition included a laser on a tripod reflecting to the top of the tall object. The students in the observer condition did not act out the problems, and the teacher did not physically act out the problems but modeled them through drawings on the board.

Procedure

The teachers worked with a researcher before the unit began to become comfortable with the curriculum. This included professional development focusing on instructional strategies such as leading class discussions, discussing and editing lesson plans, and observing the researcher teach the activities to a test group of students. To ensure that all students in all classes received similar instruction, each teacher followed the lesson plans as closely as possible throughout the unit. In the embodied condition, the teachers physically acted out their ideas and encouraged their students to do the same, but in the observer condition, they were asked to refrain as much as possible from modeling direct embodiment.

The teachers explained to their students that the unit on similarity would be part of an experiment testing new kinds of instruction. This was true for both conditions as the teachers in this study typically followed a more traditional instructional approach to teaching driven by direct instruction, while the new curriculum included very little direct instruction and was much more student centered.

The students took a pre-test at the beginning of the unit. Following two weeks of instruction, the students took a post-test and survey.

Measures and Coding

Pre- and Post-Test. I give a detailed description of pre- and post-test data in Section 2. To summarize, I measured student achievement using a pre- and post-test created based off of items in other research studies (Lamon, 1993; Misailidou &

Williams, 2003), released items from the National Assessment of Educational Progress, and from school assessments (See Appendix A). I found that students in the embodied condition had significantly greater overall learning gains than the students in the observer condition. I also found that this difference was due to a significantly larger gain on the conceptual items on the test. There were no significant differences in learning gains on the procedural items.

Survey. In order to look more closely at any differences in the learning process, I wanted to see what the most salient features of the activities were and if there were any differences by condition. To do this, I gave a survey at the end of the unit. The survey had 8 questions, one for each of the activities from the unit. For each activity, the students were given the following prompt, "Describe everything that you remember about each activity listed below (i.e. what the purpose of the activity was, what you did during the activity, who you worked with, what the specific problems were that you solved, what you learned from the activity, or anything else that you remember." The teachers who administered the survey read the prompt aloud and further emphasized that the students should write down anything that stood out to them from that activity—whether they were hungry, had trouble understanding something, or were distracted. The students had 30 minutes to complete the survey.

Coding. Students were instructed to write, "Absent" as their response to questions about activities they had missed because they were not in class. Those responses were excluded from my analyses. Since there were 8 questions, a student could have up to a total of 8 written responses.

Viewpoints. It is difficult to know exactly what viewpoint a person is taking at any given time. Past researchers have determined the viewpoints of subject participants through self-report (Nigro & Neisser, 1983) or more commonly by analyzing their speech and/or gestures (DeLiema & Enyedy, 2012; Gander, 2006; Ju & Kwon, 2007; McNeill, 1992; Ochs et al., 1994; Ochs et al., 1996; Young & Nguyen, 2002). By analyzing textbooks and teachers' talk, Young and Nguyen (2002) use first and third person verbs along with other grammatical analyses to show that textbooks use objective, third person viewpoint language that keeps the reader at a distance, while teachers construct language in such a way as to make themselves and their students active participants in science. Gander (2006) also uses pronouns to determine whether game players are using a first or third person viewpoint. This study employs a similar strategy for determining viewpoint to analyze students' descriptions of the activities in which they participated over the course of the unit.

Within each survey response, I looked at the narrative point-of-view students used in each sentence and used this as an indicator of viewpoint. I classified a sentence as "first person" if the student relayed information from her perspective using pronouns such as "I" or "we." I classified a sentence as third person if the student relayed information from an observer or non-participant perspective. This might involve using pronouns such as "it" or "they" or using nouns as the subject of the sentence such as "the ratio was 2:1." I classified a sentence as "second person" if the student used the pronoun "you."

If all of the sentences in a response were classified as first person, then the response was labeled first person. If two sentences in a response were classified as first person, and one sentence in the same response was classified as third person, then the response was labeled both first and third person. See Table 3.1 for examples.

Table 3.1. Examples of coding for point-of-view.

ID	Response	First Person	Second Person	Third Person
A	I worked with Elijah and the teacher will give us a ratio for ex. 1 to 4 and I would have to walk one for every four steps he took and they would measure us. And we couldn't move when the teacher said RED LIGHT.	X		
B	To find out how far each dot moved, we had to use fractions.	X		
C	We got in partners, and one was block 1 and the other was block 2. We had to walk and keep the same distance just like the blocks on the screen.	X		X
D	A paper in witch we Kendre, Jordan, and I all drew dot across it and we drew theyre motions and figured out ratios. MADE ME LATE FOR LUNCH.	X		
E	3 partners again reviewing on angle and side postulates while circuling around a triangle. If land on a side you are a side, if landing on a angle you are a angle. And then justify what postulate you are.		X	X
F	The purpose was to see if the triangles were similar depending on side, sizes, or angles.			X
G	It was with Willy, we measured a pole thing, I saw my friend Stephanie, I was the ratio to the pole.	X		X
H	It was just a lot of proportions and using logical thinking of math to figure out if triangles were similar.			X

If language is grounded in action (Barsalou, 1999; Glenberg, 1997; Glenberg & Robertson, 1999, 2000), then it is reasonable to assume that someone's verbal or written description about a recent memory would likely reflect that person's viewpoint while experiencing the activity. However, I am not assuming a one-to-one correspondence between first person writing and a first person viewpoint or third person writing and a third person viewpoint. I am most interested in examining the hypothesis that direct embodiment supports switching viewpoints, so I chose this classification system for viewpoint because it will be less subjective and more conservative at estimating switching viewpoints.

After labeling each of the 8 responses, I calculated the percentage of each student's responses that included sentences that were first, second, and third person. For example, Table 3.2 shows how each of a student's 8 responses were labeled. Response 1 was composed entirely of sentences written in the first person, so it is labeled first person. Response 2 was composed of one or more sentences in the first person and one or more sentences in the third person, so it is labeled both first and third person, etc. Since 4 of the 8 responses are labeled first person, this student has an overall percentage of 50% of responses including the first person. Since 6 of the 8 responses are labeled third person, the student has an overall percentage of 75% of responses including the third person.

Some students left a survey item blank or responded, "I don't know." These responses were not included in my analyses of viewpoint.

Table 3.2. Example calculations for overall percentage of responses for each point-of-view

	Response										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**% 1st**	**% 2nd**	**% 3rd**
Point-of-view Label	1st	1st & 3rd	3rd	1st	3rd	3rd	1st & 3rd	3rd	50	0	75

Switching Viewpoints. To examine the hypothesis that students in the embodied condition would be more likely to switch viewpoints during an activity, I used switching narrative points-of-view within a response as a proxy for switching viewpoints. For each response, I listed the total number of points-of-view used. For example, if all the sentences in a response were classified as first person, then that response had a total of 1 point-of view. If the first sentence of a response was classified as first person, but all the other sentences were classified as third person, then that response had a total of 2 points-of-view.

When a response included more than one point-of-view, I interpreted this as an indicator of the student likely switching or alternating between viewpoints during the activity. To gauge how often students might have switched viewpoints, I added the total number of points-of-view for each of a student's responses and divided that by the number of questions they answered. This gave me the average number of points-of-view per response for each student. For example, a student who only used one point-of-view in each response would have an average number of points-of-view of 1, but, as in Table

3.3, if a student wrote 6 responses using a single point-of-view and 2 responses using 2 points-of-view, the students would have an average number of points-of-view of 1.25.

Table 3.3. Example calculation for average number of points-of-view (Avg # PoV).

	Response									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total Points-of-view**	**Avg # PoV**
Point-of-view Label	1st	1st & 3rd	3rd	1st	3rd	3rd	1st & 3rd	3rd		
# of Points-of-view used	1	2	1	1	1	1	2	1	10	1.25

Word Count. After the teachers administered the survey to their students, more than one commented that it seemed like the students in the embodied condition were writing more than the students in the observer condition. The number of words a student writes can be used to gauge how much a student remembers about the activity, so if there was a difference in word count by condition, it would be interesting because it would indicate that one group of students remembered more about the activities.

In order to test this, I counted the total number of words students wrote and divided that by the number of questions they had the opportunity to answer. If a student was absent and missed activities, those items were not included, but if a student just left a response blank or wrote "I don't know," the response was counted as zero words. This gave me the average word count for each student.

Mathematical and Non-mathematical Details. In order to investigate any differences in the types of information students remembered, I engaged in a grounded theory analysis of the students' written responses (Glaser & Strauss, 1967; Strauss & Corbin, 1998) to analyze the content of students' responses. First, I read through the responses using open coding (Glaser & Holton, 2004; Holton, 2007) to develop an emerging list of codes that captured the details the students were describing. I made note of every recognizable detail described in the response, and I summarized what the detail was. For example, many students mentioned names of people they worked with or mentioned their group. I summarized this as partner/group. Several students also mentioned taking measurement, which I summarized as measuring. These detail summaries became codes that I then looked for in each response. That is, after one student introduced a new detail—mentioning proportions or referencing a handout—I would then look for that detail in each response. This required a variation of the constant comparative method (Glaser & Holton, 2004), as I repeatedly moved between new, unanalyzed responses and those that I had previously examined. At the conclusion of this iterative process, I went through the data a final time to ensure uniformity in my coding.

I noticed that some of the types of details were about mathematics while others were not, so I classified the codes as mathematical details or as non-mathematical details. To count as a mathematical detail, the detail had to describe a mathematical relationship, a strategy to solve a problem, a mathematical figure, a dimension used in a problem, or a mathematical theorem. Mathematical details could be mathematically correct or incorrect—I was only interested in whether the students choose to include information related to mathematics or related to other aspects of the activity or the environment.

For example, one student wrote, “had to try to figure out what was the ratio of the dots. Me, Edward, Mario.” The student mentioned “figure out” which refers to solving a problem and “ratio” which is a mathematical relationship, so this response had 2 mathematical details. The student also mentioned “dots,” which refers to a visual description of the representation in the activity, and the names of group members, which refer to partners or groups, so this response had 2 non-mathematical details.

Another student wrote, “A pair of students had to walk at a ratio.” This student also mentioned “ratio,” a mathematical detail. The student mentioned “a pair of students,” which also refers to partners or groups, and the student said they, “had to walk,” which refers to walking or moving. This response included 2 non-mathematical details. For more examples, see Table 3.4. Mathematical details are italicized and non-mathematical details are underlined.

I counted the number of mathematical and non-mathematical details for each response and found an average number of mathematical and non-mathematical details for each student. I wanted to see if there would be any differences by condition.

Table 3.4. Examples of coding for mathematical and non-mathematical details. Mathematical details are italicized and non-mathematical details are underlined.

Label	Response	# Math Details	Math Detail Codes	# Non-Math Details	Non-Math Detail Codes
A	I worked with Elijah and the teacher will give us a *ratio* for ex. *1 to 4* and I would have to walk *one for every four* steps he took and they would *measure* us. And we couldn't move when the teacher said RED LIGHT.	4	ratio, example, for every, measure	7	partner/group, teacher, given/assigned, walking/moving, steps, stopping, red light
B	To *find out* how *far* each dot moved, we had to use *fractions*.	3	calculate, distance, strategy	2	dots, walking/moving
C	We got in partners, and one *was* block 1 and the other was block 2. We had to walk and keep the *same distance* just *like the blocks on the* screen.	3	representation, equality, distance, connection to previous activity	4	partner/group, blocks, walking/moving, screen
D	A paper in witch we Lauren, Jordan, and I all *drew* dot across it and we drew theyre *motions* and *figured* out *ratios*. MADE ME LATE FOR LUNCH.	4	drawing, moving/walking, calculate, ratio	4	handout, partner/group, dots, environment
E	3 partners again reviewing on *angle and side postulates* while circuling around a *triangle*. If land on a *side* you *are* a side, if landing on a *angle* you are a angle. And then *justify* what postulate you are.	6	postulates/theorems, triangle, side, representation, angle, justification/proof	2	partner/group, walking/moving
F	The purpose was to *see* if the *triangles* were *similar* depending on *side, sizes,* or *angles*.	5	calculate, triangle, similarity, side, strategy	0	
G	It was with Ezra, we *measured* a pole thing, I saw my friend Ladaun, I *was* the *ratio* to the pole.	3	measuring, representation, ratio	3	partner/group, tall object, environment
H	It was just a lot of *proportions* and *using logical thinking* of math to *figure out* if *triangles* were *similar*.	5	proportions, strategy, calculate, triangles, similar	0	

Results

Section 2 reports on the pre- and post-test findings for this study. I wanted to further investigate my findings that students in the embodied condition had greater learning gains than students in the observer condition by looking at any differences in students' post-unit reflections on the activities.

Viewpoints

I predicted that students in the embodied condition would be more likely to adopt a first person viewpoint during the activities because they were directly embodying mathematics concepts. Thus, I thought those students would have a higher average percentage of responses written from a first person narrative point-of-view indicating that they had experienced activities more often from a first person viewpoint. I also predicted that students in the observer condition would be more likely to view their activities from a third person viewpoint since they were not participating in direct embodiment. I thought these students would have a higher percentage of responses written from a third person narrative point-of-view showing they had experienced activities primarily from a third person perspective.

I conducted a two-way repeated measures ANOVA on overall percentage of each perspective with a Greenhouse-Geisser correction, a between subjects factor of condition (embodied, observer), and a within subjects factor of perspective (first, second, third). This would reveal any differences between the average percentages of responses in each perspective between conditions. There was a significant difference overall in the use of first, second, and third person, $F(1.63, 237.67) = 106.85$, $MSE = 0.13$, $p < .001$. Students were significantly more likely to use a first person (57.7%) or third person (59.5%) perspective than a second person point-of-view (11.1%). There was also a significant main effect of point-of-view, $F(1, 146) = 16.87$, $MSE = 0.017$, $p < .001$. Students in the embodied condition had a significantly higher percentage of first person and second person usage than students in the observer condition. There were no significant differences between the percentages of third person use for the two conditions (See Figure 3.17).

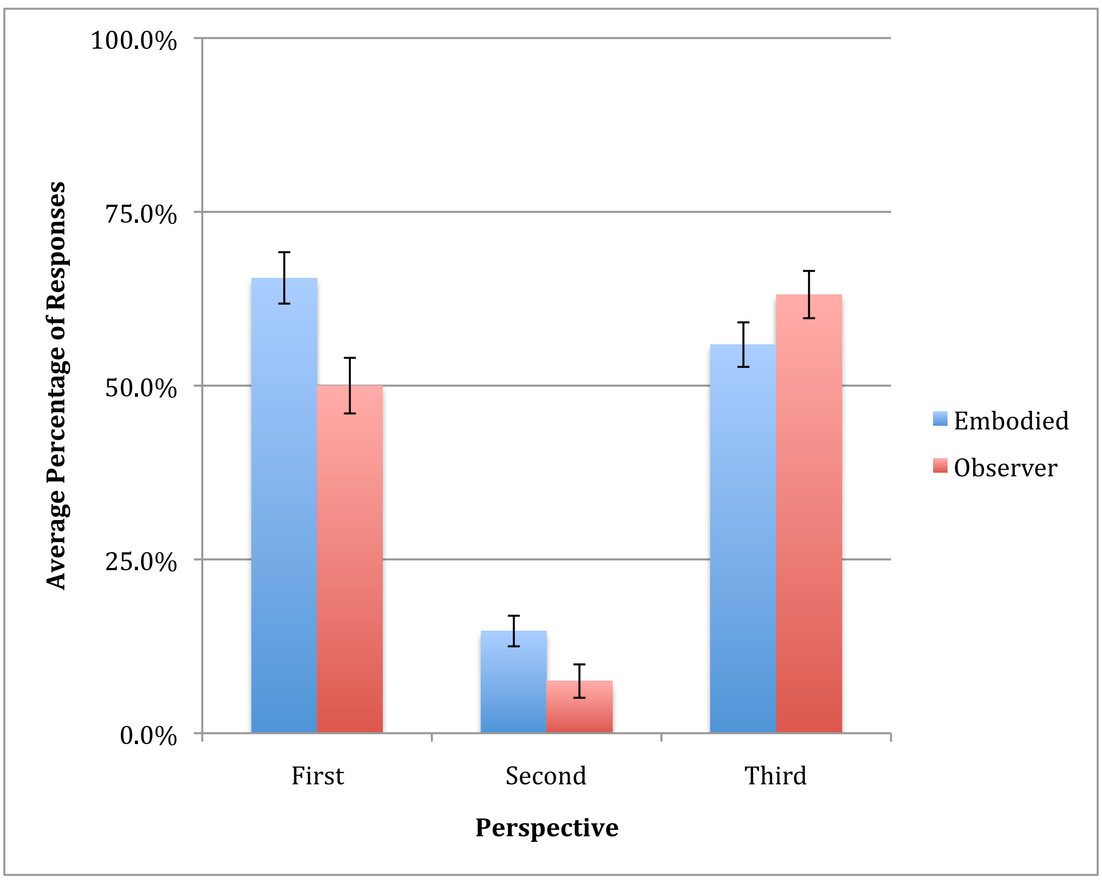


Figure 3.17. Average percentage of responses from each point-of-view by condition.

Switching Viewpoints

I conjectured that students in the embodied condition would be more likely to switch viewpoints during the activities than students in the observer condition and thought that this would be reflected in their written survey responses by alternating narrative points-of-view. First, I conducted a chi-square test of independence to examine the relation between condition and whether or not a student switched points-of-view at least once in one of their responses. The relationship between these variables was significant, X^2 (1, 148) = 10.19, $p < .01$. Students in the embodied condition were more likely to switch points-of-view in their writing at least once than students in the observer condition (See Table 3.5).

Table 3.5. Switching points-of-view by condition.

	Never Switched	Switched at Least Once	Total
Observer	24	45	69
Embodied	10	69	79
Total	34	114	148

I also conducted a one-way ANOVA on the average number of points-of-view used with a between subjects factor of condition (embodied, observer). There was a significant effect of condition on average number of points-of-view used, $F(1, 146) = 17.56$, $p < .001$. Students in the embodied condition employed a significantly higher average number of points-of-view (M = 1.36, SD = 0.24) than students in the observer condition (M = 1.20, SD = 0.21).

Word Count

I predicted that students in the embodied condition would write more than students in the observer condition because they would remember more about the activities. I conducted a one-way ANOVA on average word count per response with a between subjects factor of condition (embodied, observer). There was a significant effect of condition on average word count, $F(1, 146) = 28.76$, $p < .001$. The average number of words per response was significantly higher for the embodied condition (M = 22.54, SD = 8.72) than for the observer condition (M = 15.01, SD = 8.31) (See Figure 3.18).

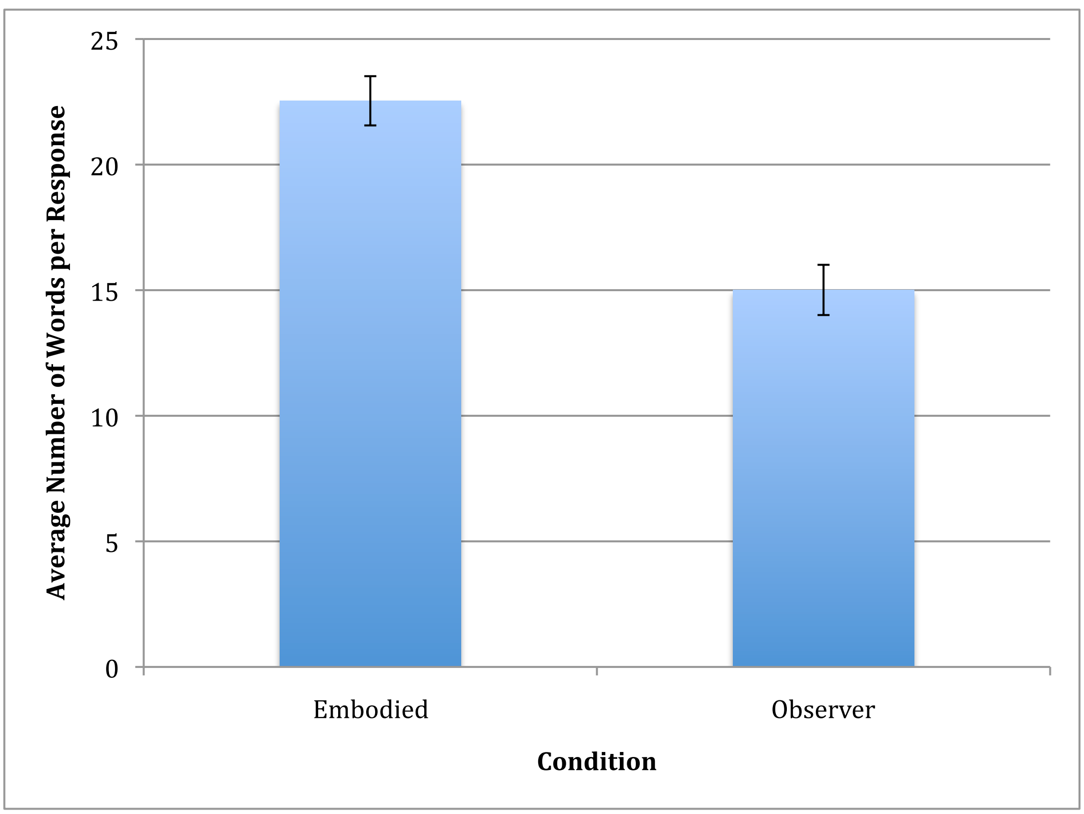


Figure 3.18. Average word count per response by condition.

Mathematical and Non-Mathematical Details

To examine if there were differences in the features of each activity that stood out to students, I compared whether there were be differences by condition in the number of mathematical and non-mathematical aspects of the activities. I conducted a two-way repeated measures ANOVA with a between subjects factor of condition (embodied, observer) and a within subjects factor of detail type (mathematical, non-mathematical). There was a significant main effect of detail type, $F(1, 146) = 62.28$, $MSE = 0.85$, $p < .001$. Overall, students wrote significantly more mathematical details per response (M = 2.35, SE = 0.10) than non-mathematical details (M = 1.51, SE = 0.06). There was also a significant main effect of condition, $F(1, 146) = 30.81$, $p < .001$. Post hoc testing revealed that students in the embodied condition wrote significantly more mathematical and non-mathematical details than students in the observer condition (See Figure 3.19).

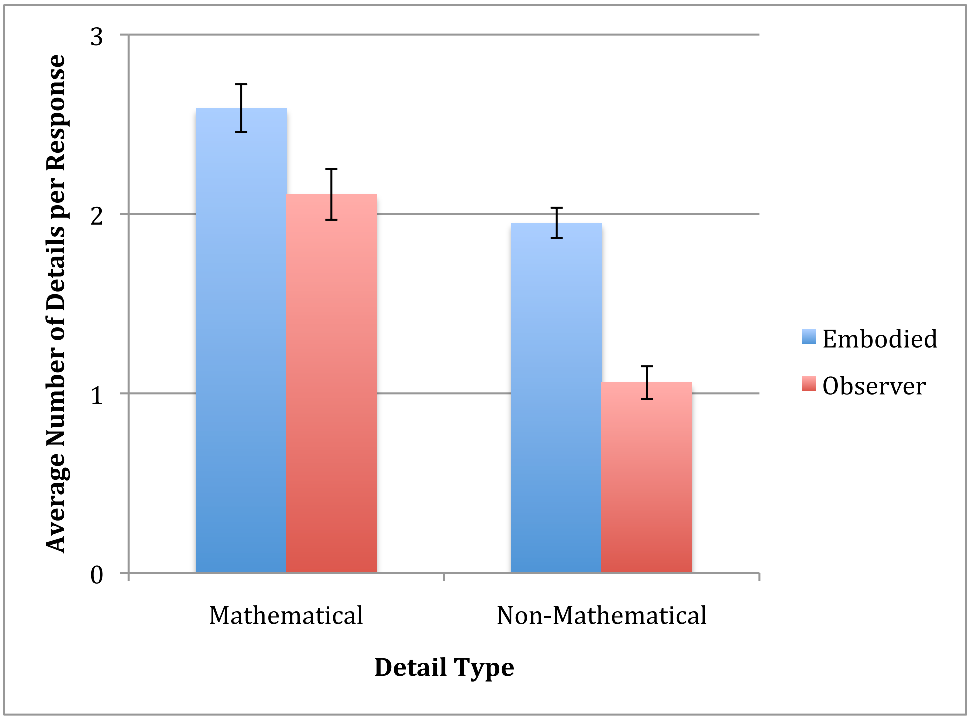


Figure 3.19. Average number of mathematical and non-mathematical details per response by condition.

Discussion and Conclusion

These findings indicate that students who learn through direct embodiment are significantly more likely to use a first person point-of-view when writing reflections on their learning. There are a few possible reasons for this. First, it could be that students in the embodied condition were more likely to see themselves as active participants in their learning, so they were more likely to write using themselves as the subject of their sentences. I agree that the students in the embodied condition were certainly more physically active than the students in the observer condition, and this could have some impact on the point-of-view that students used. At the same time, I designed the activities so that students in both conditions had similar amounts of control or similar abilities to manipulate objects. For instance, in the Growing & Shrinking Triangles activity, the embodied condition created triangles with their bodies, and the observer condition created triangles by drawing. Because of this, I see several cases of students in the observer condition writing from a first person point-of-view, such as this response D describing Activity 3:

> A paper in witch [*sic*] we Lauren, Jordan, and I all drew dot across it and we drew theyre motions and figured out ratios. MADE ME LATE FOR LUNCH.

Another, possibly more compelling reason for this difference in point-of-view is that the students in the embodied condition were directly embodying mathematics and therefore more likely to be using a first person viewpoint during the activity, which translated into writing about the events from a first person point-of-view. An example of this comes from response G:

> …I was the ratio to the pole.

It is likely that both of these reasons played a part in this difference in narrative point-of-view.

The finding that I found most interesting was that students in the embodied condition were more likely to switch points-of-view in their writing. In English and language arts classes, students are taught to establish one or multiple points-of-view in their writing depending on their intent (National Governors Association Center for Best Practices, 2010). I think switching points-of-view in writing means it was likely that students switched viewpoints during the activity. For example, the following response is written from two points-of-view:

> 3 partners again reviewing on angle and side postulates while circuling [*sic*] around a triangle. If land on a side you are a side, if landing on a angle you are a angle. And then justify what postulate you are. (response E)

The student starts out writing from the third person describing three partners and what the goal of the activity is. This student has a third person viewpoint in this case. The student is one of the three partners referred to but is writing as if he is outside of the action, watching. Then the following two sentences are written from a second person point-of-view. The student uses the pronoun "you" in a similar manner to Ochs et al.'s (1996) "indeterminate" point-of-view, which blurs the line between the student as a mathematician and the student as the mathematics. This student is drawing a relationship between himself and the sides and angles of the triangle during the activity. Given the two distinct viewpoints represented in the single response, I take this as evidence that the student switched viewpoints during the activity.

A second example also shows switching:

> We got in partners, and one was block 1 and the other was block 2. We had to walk and keep the same distance just like the blocks on the screen. (response C)

This student starts by writing in the first person ("We got in partners"). Then the student switches to a third person point-of-view ("one was block 1 and the other was block 2"). Finally, the student switches back to the first person point-of-view ("We had to walk and keep the same distance…").

I take the finding that students in the embodied condition were more likely to switch points-of-view than students in the observer condition as support for my hypothesis that direct embodiment affords switching viewpoints. The next step in my research is to examine the videotaped interactions of students during the learning activities to see how frequently switching actually occurs.

I also find it very interesting that the embodied condition wrote significantly more than the observer condition. I have two hypotheses for why this may be. First, it could be that aspects of the direct embodiment activities made it easier for students to construct richer memories of the entire episode. The direct embodiment activities were quite different from the students' traditional instruction, and included lots of variety in the kind of physical activity required. The activities included Wii remotes, walking with a partner in an open room, creating triangles with measuring tapes, and going outside to measure a tall object. These various scenarios in themselves might be enough to help students produce richer memories, giving students more "hooks" or mental cues to use to help them recall the activity. In contrast, students in the observer condition sat at their desks during each activity, and the only physical movement came when they arranged their desks together to work in a group or with a partner.

It could also be that the students in the embodied condition wrote more because the direct embodiment activities prompted students to reflect more than the more passive activities. Translating between first person and third person viewpoints could take more mental effort and reflection to determine how information from one viewpoint fits in with another. If students spent more time reflecting and mentally assimilating information from different viewpoints, this could have lead students to write more. Since I have evidence that students in the embodied condition were more likely to switch viewpoints, it could be that this lead to longer written responses on the survey.

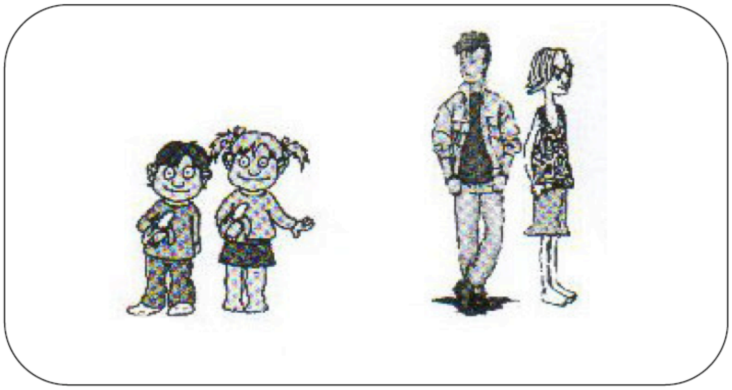
If the first hypothesis was true, and students in the embodied condition remembered more because of the variety of experiences they had, then I would think that their survey responses would have included more information describing the activities or the environment (i.e. non-mathematical details) than information about the actual mathematics. As it turns out, even though students in the embodied condition included significantly more non-mathematical details than the observer condition, both conditions wrote significantly more mathematical than non-mathematical details. It could be that the embodied condition used the activity details to help them recall more mathematical details, but it seems likely that the direct embodiment activities prompted students to reflect more during the activities, helping them remember it better when they recalled them later.

The findings presented here suggest that learners can benefit from activities or environments that afford translation among multiple viewpoints. Finding ways to incorporate direct embodiment activities in the classroom may make mathematics more accessible to students by increasing possible points of entry. I am curious about my finding that students in the embodied condition seemed to remember more about the activity and the mathematics than students in the observer condition, and I plan to investigate this further in the future. Particularly, I would like to look at whether the richness and the variety in the activities prompted greater recall or whether taking multiple viewpoints prompts students to reflect more.

Appendix A

Pre- and Post-Test

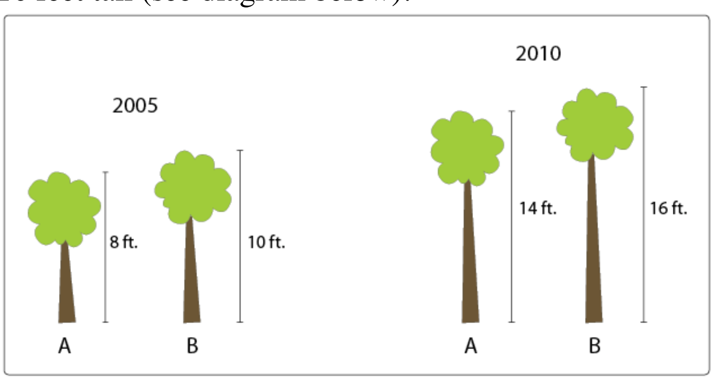
1. The picture below shows Amy and Richard when they were young on the left and as they are now on the right.



a. Who grew faster between the first and second pictures? Amy or Richard?

b. Explain your reasoning for the answer you gave in part a.

2. In 2005, tree A was 8 feet tall and tree B was 10 feet tall. In 2010, tree A was 14 feet tall and tree B is 16 feet tall (see diagram below).



a. Over the five-year period, which tree's height increased the most proportionally?

b. Explain your reasoning for the answer you gave in part a.

3. This is a map of an area in Houston.

a. What does the box in the lower right hand corner mean?

b. Give an example of how you would use the information in the box on the lower right hand corner.

4. An onion soup recipe for 8 people is as follows:

8 onions
4 cups water
4 chicken soup cubes
8 tablespoons butter
½ cup cream

To make the recipe for 6 people, how much cream do you need? Show your work for credit.

5. Only one of these players can be selected for the school basketball team.

   Matt scored 14 goals out of 20 attempts.

   Henry scored 18 goals out of 25 attempts.

   Which one is the better shooter? Justify your answer to receive credit.

6. Over the course of a basketball season, Jane's scoring average improved from 25 to 40 points per match. Emma's scoring average improved from 30 to 45 points per match.

   a. Which girl improved the most over the course of the season?

   b. Explain how you came to your answer in part a.

7. The figure above shows two right angles. The length of AE is x and the length of DE is 40. Show all of the steps that lead to finding the value of x. Your last step should give the value of x.

8. Which of the following pairs of geometric figures must be similar to each other?

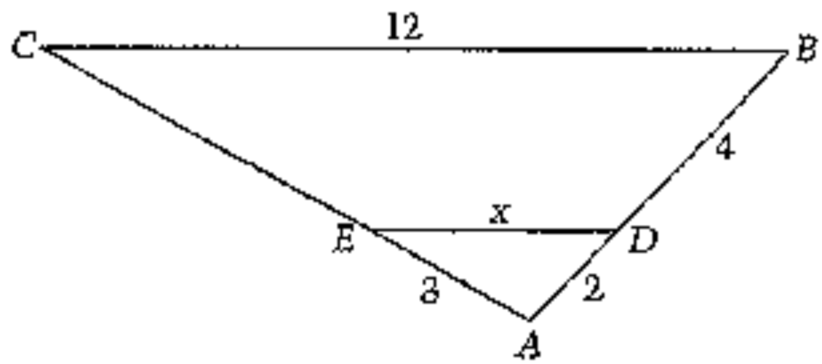
A. Two equilateral triangles

B. Two isosceles triangles

C. Two right triangles

D. Two rectangles

E. Two parallelograms



9. If triangles *ADE* and *ABC* shown in the figure above are similar, what is the value of x?

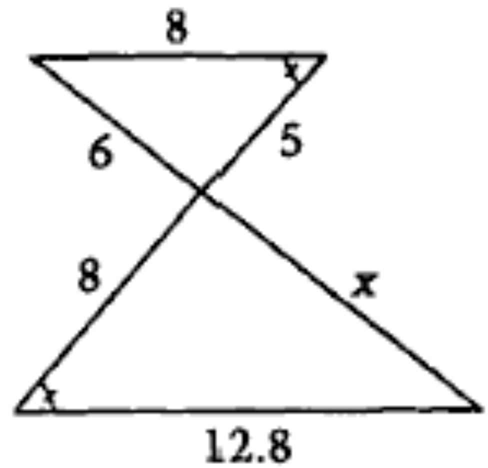
A. 4

B. 5

C. 6

D. 8

E. 10



10. In the figure above, the two triangles are similar. What is the value of x?

11. What is the ratio of the length of a side of an equilateral triangle to its perimeter?

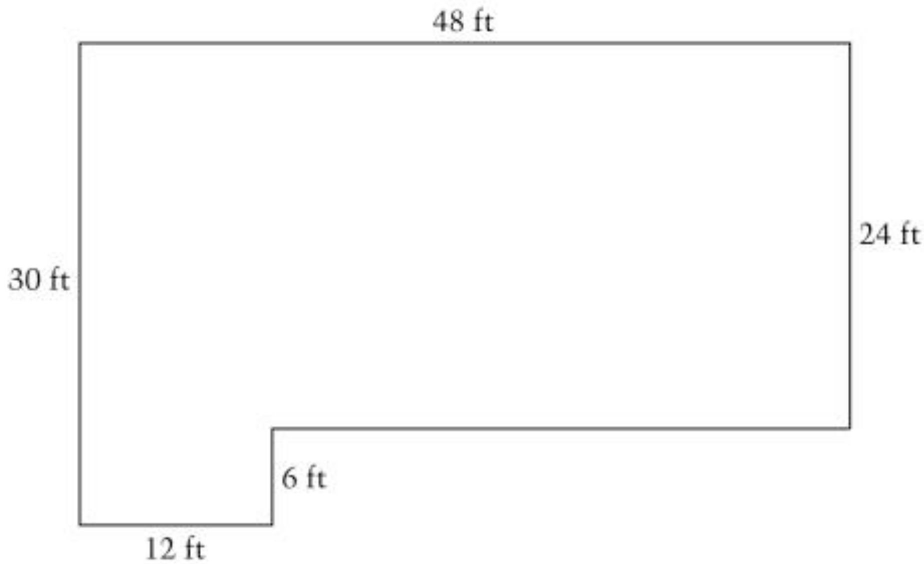
    A. 1:1

    B. 1:2

    C. 1:3

    D. 2:1

    E. 3:1



Use the figure above for #12 and 13.

12. Use a ruler to find, in terms of inches and feet, what scale has been used to construct the diagram.

13. If you were to redraw the diagram using a scale of $\frac{3}{4}$ inch = 10 feet, what would be the length of the side that is 48 feet?

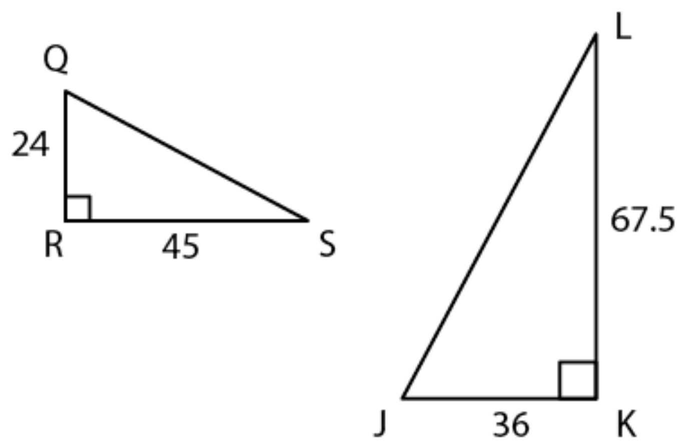
A. 3.0 in

B. 3.6 in

C. 5.6 in

D. 7.5 in

E. 12.0 in

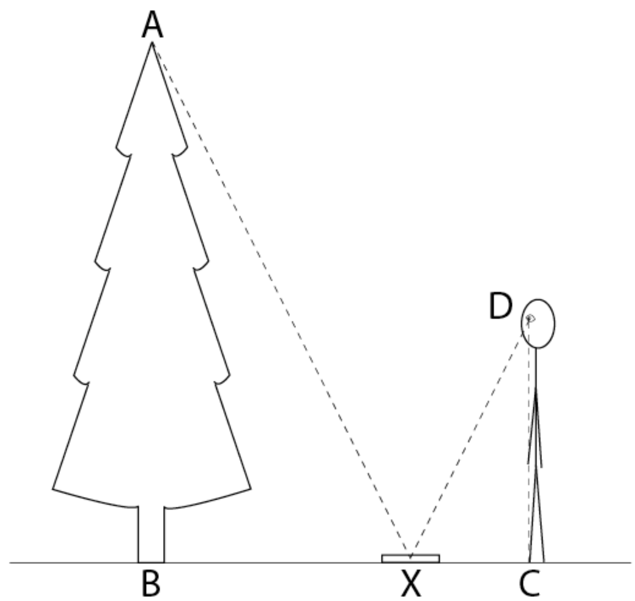


14. a. Are the two polygons above similar?

b. Give an argument to support your answer to part a.

c. If the triangles are similar, write the similarity ratio and a similarity statement.

15. The ratio of the side lengths of a quadrilateral is 6:2:3:7, and its perimeter is 126 meters. What is the length of the shortest side?



16. a. Antonio is about 5 feet tall. He is trying to measure the top of a tree. He places a mirror on the ground 4 feet way from the tree at point X. Then he plans to walk back in a straight line to a point C where he can look at the mirror and see the reflection of the top of the tree. If Antonio guesses that the tree is about twice as tall as he is, about how far away from the mirror should he walk so that he will likely be able to see the reflection of the top of the tree in the mirror? In other words, how long should the distance be between point X and point C?

b. Explain how you got your answer to part a.

17. Sketch and label two triangles that fit the following description:

- ΔABC and ΔTRS are similar.
- The ratio of the side lengths of ΔABC to the side lengths of ΔTRS is $\frac{5}{6}$.

Label the side lengths and label the angles with measurements that you make up that fit this description.

Appendix B

Lesson Plans

Lesson 1: Ratio & Proportion

Lesson Objectives:

- Students will use proportions to make predictions and solve problems.
- Students will collect data in order to understand proportional relationships.
- Students will apply knowledge of proportional relationships to solve problems in different contexts.
- Students will distinguish between proportional and additive relationships.

Target Grade Level:

- High school geometry

TEKS:

- G.5.B Geometric Patterns: use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.
- G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures.

Embodied Materials Needed: Wii remotes, infrared sensor bar, OSCulator, Max/MSP, Make the Screen Green Program, Handout

Observer Materials Needed: PowerPoints, Large paper for posters, markers, markers, Student handouts, white boards, dry erase markers

Instructional Sequence:

1. Activity 1—Make the Screen Green

- *Embodied Version*
    1. Engagement – Capture student interest by discussing Wii remotes. Ask the following questions:
        - i. Has anyone ever played with the Wii before?
        - ii. Who can describe how Wii remotes work? (The Wii remotes have a built in accelerometer and infrared detection, so when one of them is pointed at the infrared sensor bar, it can sense the position of the remote and how fast it is moving.)

iii. Demonstrate how the Wii remotes control the two blocks on the screen.

2. Ask for a volunteer to try to use the Wii remotes to make the screen turn green.
3. As the student tries to make the screen green, ask the class:
   i. What is the hidden rule that you think makes the screen green?
   ii. What do you think makes the screen to red?
4. After the screen has turned green at least once, ask students to discuss with a partner what makes the screen turn green. Then have them record their initial hypothesis for the hidden rule on their handout. Have students share their ideas with the class.
   a. Ask students to use evidence to defend their idea.
   b. Use your hands to model a particular configuration of the blocks and ask the students to predict whether the screen would be red or green and why.
5. Direct Embodiment – Ask students to use their hands to show a partner what they think does or does not make the screen green.
6. Discuss – Have several students share what they think makes the screen green. Prompt them to use their hands to mimic the blocks.
7. Have several students take turns using the Wii remotes to try to make the screen turn green.
8. Every few minutes, have students turn their partners and describe what they think makes the screen turn green. Ask them to use their hands to help them describe their ideas.
9. Display the lines on the screen. Have more students test out their ideas.
   a. How can you use the numbers to help you make the screen turn green?
   b. Can you use numbers to describe the hidden rule that makes the screen green?
10. Have students record what they think the hidden rule is on their handout. Ask a few students to share their second hypothesis with the class:
    a. Ask students to use evidence to defend their ideas.
    b. Use your hands to model a particular configuration of the blocks and ask the students to predict whether the screen would be red or green and why.
11. Other discussion questions to ask:
    a. What evidence do you have to support your hypothesis?
    b. What are different ways to describe the relationship between the blocks that makes the screen turn green?
    c. Is it possible to move the blocks from the bottom to the top and keep the screen green the entire time? Why or Why not?
12. Have students answer question #3 in their handout.

- *Observer Version*
  1. Engagement – Capture student interest by telling them they will have to find a hidden rule for a program that makes the screen turn green under certain circumstances. Ask the following questions:
  2. Ask for a volunteer to try to determine the hidden rule that makes the screen green.
  3. As the student watches video 1, ask the class:
     i. What is the hidden rule that you think makes the screen green?
     ii. What do you think makes the screen to red?
  4. After the screen has turned green at least once, ask students to discuss with a partner what makes the screen turn green. Then have them record their initial hypothesis for the hidden rule on their handout. Have students share their ideas with the class.
     a. Ask students to use evidence to defend their idea.
     b. Verbally describe a particular configuration of the blocks and ask the students to predict whether the screen would be red or green and why.
  5. Ask students to verbally describe to a partner what they think does or does not make the screen green.
  6. Discuss – Have several students share what they think makes the screen green.
  7. Ask the whole class to think about what makes the screen turn green. Play video 2. Pause or rewind the video when students want to test certain hypotheses.
  8. Every few minutes, have students turn their partners and describe what they think makes the screen turn green.
  9. Play video 3 which displays the lines on the screen. Have more students test out their ideas by pausing and rewinding the video to certain points.
     a. How can you use the numbers to help you make the screen turn green?
     b. Can you use numbers to describe the hidden rule that makes the screen green?
  10. Have students record what they think the hidden rule is on their handout. Ask a few students to share their second hypothesis with the class:
      a. Ask students to use evidence to defend their ideas.
      b. Verbally describe a particular configuration of the blocks and ask the students to predict whether the screen would be red or green and why.
  11. Other discussion questions to ask:
      a. What evidence do you have to support your hypothesis?
      b. What are different ways to describe the relationship between the blocks that makes the screen turn green?
      c. Is it possible to move the blocks from the bottom to the top and keep the screen green the entire time? Why or Why not?
  12. Have students answer question #3 in their handout.

2. **Activity 2—Ratio Red Light Green Light/Ratio Race**
   - *Embodied Version*
     1. Explain – The class just experienced proportional relationships by moving their hands. Now they will experience proportional relationships while moving their bodies in a game of Ratio Red Light Green Light. Ask students if they have played the game before. Warn that the rules in this game are slightly different.
     2. Partners – Put students in groups of 2 (or 3 if necessary), and have each student choose a letter A or B.
     3. Explain: Show the Ratio Red Light/Green Light PowerPoint, and use it to explain the game. You will need open space for students to walk in a straight line.
        - Teacher gives target ratio. Example- 5:1
        - This means that the ratio of Student A's distance to the starting line to Student B's distance to the starting line must always be 5:1.
        - Partners will practice for about 2 minutes.
        - Game begins.
        - When teacher says, "Green Light," students move forward. They must maintain the correct ratio of their distances.
        - When teacher says, "Red Light," students freeze.
        - Students mark their positions.
        - They measure their distances and compute their ratio on their WS.
        - Each pair shares their ratio, and the class determines who was the closest to the target ratio.
     4. Direct Embodiment Round 1 - The ratio of Student A's distance to the starting line to Student B's distance to the starting line is 2:1.
     5. Discuss how you can tell which pair has the closest ratio to 2:1. Each pair of students comes up with their own strategy. Have one or two pairs of students share their strategies with the class.
     6. Direct Embodiment Round 2 - The ratio of Student A's distance to the starting line to Student B's distance to the starting line is 1:4.
        - Note: As students are working, go around and listen to understand the strategy each pair is using. Make a mental note of different strategies to bring up in discussion later.
     7. Direct Embodiment Round 3 - The ratio of Student A's distance to the starting line to Student B's distance to the starting line is 3:2.
     8. Class discussion – Ask the following questions:
        - What strategies did you use? [some might include taking steps of different sizes at the same rate, or taking the same size steps at different rates]
        - Which strategies are similar? Different? How do you know?
        - What are the weaknesses of the strategies? Strengths?
        - Would your strategy work if we were in a much larger space (e.g. on the football field)? Why or Why not?

- *Observer Version*
  1. Explain – The class just visualized proportional relationships by watching the video. Now they will visualize proportional relationships through a Ratio Race.
  2. Partners – Put students in groups of 2 (or 3 if necessary).
  3. Explain: Show the Ratio Race PowerPoint, and use it to explain the game.
     - The dots have a target ratio. Example- 5:1
     - This means that the ratio of the red dot's distance to the starting line to the yellow dot's distance to the starting line must always be 5:1.
     - Watch a practice race.
     - When the race starts, the dots move forward. They must maintain the correct ratio of their distances.
     - When the race ends, the dots stop moving where they are.
     - Students estimate their distance to the starting lines and use this to compute a ratio on their WS.
     - Each pair of students determines whether or not the pair of dots maintained the target ratio.
  4. Round 1 - The ratio of the red dot's distance to the starting line to yellow dot's distance to the starting line is 2:1.
  5. Discuss how you can tell how close the ratio to 2:1. Each pair of students comes up with their own strategy. Have one or two pairs of students share their strategies with the class.
  6. Round 2 - The ratio of the red dot's distance to the starting line to yellow dot's distance to the starting line is 1:4.
     - Note: As students are working, go around and listen to understand the strategy each pair is using. Make a mental note of different strategies to bring up in discussion later.
  7. Round 3 - The ratio of the red dot's distance to the starting line to the yellow dot's distance to the starting line is 3:2.
  8. Class discussion – Ask the following questions:
     - What strategies did you use?
     - Which strategies are similar to other students'? Different? How do you know?
     - What are the weaknesses of the strategies? Strengths?
     - Would your strategy work if the screen were much larger (e.g. on the football field)? Why or Why not?

3. Homework – Scenario 1

4. Activity 3 – Who Went Further?

- *Embodied Version*
  1. Display the "Who Went Further" section of the PowerPoint found at the end of the Ratio Red Light Green Light PowerPoint.
  2. Students work with their partner. Each pair of students makes two moves according to the directions on the screen. They must try to determine who moved further on the second move.
     - Note: For this activity, all students must take steps that are the same size. Have them use the tiles on the floor to help measure their steps.
  3. Direct Embodiment: Students follow the directions on the slide and act out the two moves. Move 1- Student A takes 2 steps, and student B takes 4 steps. Move 2 – Student A takes 4 steps, and student B takes 4 steps.
  4. Discuss – On the second move, which student went further? Why?
     a. If everybody says that they moved the same amount, ask, "Let's think of the problem in a different way. Can anyone think of a reason their second moves are different?"
  5. Direct Embodiment: Have students act out the problem again.
  6. Discuss – Say, "I'm going to change my question a little. Which students moved further in relation to their original move?"
     a. Have students discuss this with their partner.
     b. Have pairs share answers with the class.
     c. Have students compare what is the same or different about each way of thinking.
     d. Note: When you think proportionally, then Student A moves three times their original amount on the second move, so in that sense, Student A moves further. When you think linearly, both students move 4 steps on their second move.
     e. Discuss how proportional thinking is different than looking at the linear distance.
- *Observer Version*
  1. Display the "Who Went Further" section of the PowerPoint found at the end of the Ratio Race PowerPoint.
  2. Students work with their partner. Each pair of students watches the dots on the screen. They must try to determine which dot moved further on the second move.
  3. Play the dots' movements. Move 1- Red dot moves 2 units, and yellow dot moves 4 units. Move 2 – Red dot moves 4 units, and yellow dot moves 4 units.
  4. Discuss – On the second move, which dot went further? Why?
     a. If everybody says that they moved the same amount, ask, "Let's think of the problem in a different way. Can anyone think of a reason their second moves are different?"
  5. Play the dots' movements again.

6. Discuss – Say, "I'm going to change my question a little. Which dot moved further in relation to its original move?"
    a. Have students discuss this with their partner.
    b. Have pairs share answers with the class.
    c. Have students compare what is the same or different about each way of thinking.
    d. Note: When you think proportionally, then the red dot moves three times its original amount on the second move, so in that sense, red dot moves further. When you think linearly, both students move 4 steps on their second move.
    e. Discuss how proportional thinking is different than looking at the linear distance.

4. Activity 4 – Ratio Red Light Green Light Challenges/Ratio Race Challenges

- *Embodied Version*
    1. Have students work in groups of three to complete the challenges on their handout.
        a. Students begin by acting out the problems and then must move to more mathematical strategies when the numerical structure becomes more complex.
    2. Have students share their strategies with the class and discuss.
- *Observer Version*
    3. Have students work in groups of three to complete the challenges on their handout.
        a. Students begin drawing pictures to solve problems and then must move to more mathematical strategies when the numerical structure becomes more complex.
    4. Have students share their strategies with the class and discuss.

5. Mini-Lesson

1. Display Mini-Lesson 1 PowerPoint.
2. Define ratio and describe different kinds.
3. Have students work problem 1 on their white boards.
4. Define proportion.
5. Ratio rearrange activity on white boards.
6. Have students work problems 2 and 3 on their white boards.

6. Homework – Complete last Challenge

Lesson 2: Ratio & Similar Figures

Lesson Objectives:

- Students will develop a definition of similar figures.

- Students will create representations of similar triangles.
- Students will apply knowledge of similarity to solve problems in different contexts.

Target Grade Level:
- High school geometry

TEKS:
- G.5.B Geometric patterns: use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.
- G.11.A Similarity and the geometry of shape: use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.
- G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures.

Embodied Materials Needed: Measuring Tape Triangles, Worksheet, Problem Solving 7-2 WS, PowerPoint

Observer Materials Needed: Growing and Shrinking Triangles WS, Problem Solving 7-2 WS, PowerPoint, Rulers, and Protractors

Instructional Sequence:

1. Activity 5 – Growing & Shrinking Triangles
- *Embodied Version*
    1. Students work in groups of 3. Each group gets a measuring tape triangle (made out of three retractable measuring tapes attached together).
    2. Show Measuring Tape Triangles PowerPoint. Have students work through steps 1-5.
        a. Direct Embodiment: First students create a triangle with their group members.
        b. Students move to create another triangle that is larger but maintains the same shape.
        c. Students create a triangle with sides that are twice as long as their original triangle. They note what happens to the angles of their triangle as it grows.
        d. Students create a triangle with sides that are three times as long as their original triangle. They note what happens to the angles of their triangle as it grows.

e. Students discuss and record their strategies for maintaining a triangle that is the same shape as it grows or shrinks.
f. Questions to ask:
    i. How must you move so that the triangle maintains the same shape? Why does that work?
    ii. As the triangle gets bigger, but the angles stay the same, what is happening to the sides?
    iii. Do all the sides increase at the same rate?
g. Give definition of similarity. Show how to write a similarity statement.
h. Direct Embodiment: Students work on #4-12 on their worksheet in their groups. They will directly embody problems 4 and 5.
i. As a class discuss strategies for #6 or 7.

- *Observer Version*
  1. Students work in groups of 3. Each student has a protractor and ruler.
  2. Show Growing and Shrinking Triangles PowerPoint. Have students work through steps 1-11.
     a. First students create a triangle on their paper.
     b. Students draw another triangle that is larger but maintains the same shape.
     c. Students draw a triangle with sides that are twice as long as their original triangle. They note what happens to the angles of their triangle as it grows.
     d. Students draw a triangle with sides that are three times as long as their original triangle. They note what happens to the angles of their triangle as it grows.
     e. Students discuss and record their strategies for maintaining a triangle that is the same shape as it grows or shrinks.
     f. Questions to ask:
        i. What do you do to draw a triangle that maintains the same shape as the original? Why does that work?
        ii. As the triangle gets bigger, but the angles stay the same, what is happening to the sides?
        iii. Do all the sides increase same way?
     g. Give definition of similarity. Show how to write a similarity statement.
     h. Students work on #11-14 on their worksheet in their groups.
     i. As a class discuss strategies for #13 or 14.

2. Homework – Problem Solving 7-2 WS

Lesson 3: Triangle Similarity AA, SSS, SAS

Lesson Objectives:

- Students will prove certain triangles are similar by using AA, SSS, and SAS.
- Students will use triangle similarity to solve problems.

Target Grade Level:

- High school geometry

TEKS:

- G.11.A Similarity and the geometry of shape: use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.
- G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures.
- G.11.B Similarity and the geometry of shape: develop, apply, and justify triangle similarity relationships.

Embodied Materials Needed: Problem Solving 7-2 WS, XYZ floor triangles, Musical Triangles PowerPoint, Problem Solving 7-3 WS

Observer Materials Needed: Problem Solving 7-2 WS, PowerPoint, Triangles WS, colored pencils, Problem Solving 7-3 WS

Instructional Sequence:

1. Discuss homework – Problem Solving 7-2 WS

2. Opening Discussion

   1. Show slides 2-4 which depict two triangles with angle and side measurements labeled (the corresponding angles are congruent and the corresponding sides are proportional). Ask students:
      a. What is the minimal amount of information needed to show that the two triangles are similar?
      b. In other words, what information could we take away and still know that the triangles were similar?
      c. What do you remember from the Growing & Shrinking Triangles activity that you could apply here?
   2. Have students discuss with a partner or small group.
   3. Students share ideas with the class.
   4. Present the 3 similarity postulates and connect them with the ideas the students came up with.
   5. Compare and contrast with the congruence postulates from previous unit.

3. Activity 6 – Musical Triangles/Similar Triangles

- *Embodied Version*
  1. Show the instructions on the PowerPoint for Musical Triangles. Explain each step and have a group demonstrate.
     a. The object of the activity is for students to use a similarity postulate to show that the triangle on the floor in front of them is similar to the triangle on the screen.
     b. The screen will give the students information they can use (e.g. angle A is congruent to angle X and angle B is congruent to angle Y).
     c. Students walk around the triangle and look at the screen to determine which postulate they will use.
     d. Direct embodiment: Then when directed, they move to stand on the appropriate location of their triangle to represent the angle or side on the screen that will help them prove the triangles are similar. Students who are sides extend their arms to the side to form a line. Students who are angles extend their arms in front of them to create an angle.
  2. Students work in groups of 3. Each group has an open space and a cardboard triangle on the floor.
  3. Play several rounds of the game. After each round, ask the following questions:
     a. Is there a similarity postulate that proves these two triangles are similar? If so, what is it? If not, what other information would you need and why?
     b. Why did you decide to stand where you are?
     c. Is there another way that you could have stood on the triangle that would be correct?
- *Observer Version*
  1. Show the instructions on the PowerPoint for Similar Triangles. Explain each step and have a student demonstrate on the board.
     a. The object of the activity is for students to use a similarity postulate to show that the triangle on the paper in front of them is congruent to the triangle on the screen.
     b. The screen will give the students information they can use (e.g. angle A is congruent to angle X and angle B is congruent to angle Y).
     c. Students look at their triangle and look at the screen to determine which postulate they will use.
     d. Then when directed, students color the appropriate parts of the triangle on their paper that will help them prove the triangles are similar. For example, if the best postulate to use is AA, then the student will color two angles on their paper.
  2. Students work in groups of 3. Each group has their own handout and colored pencils to share.
  3. Have students complete several questions. After each one, ask the following questions:

a. Is there a similarity postulate that proves these two triangles are similar? If so, what is it? If not, what other information would you need and why?
b. Why did you decide to color what you did?
c. Is there another way that you could have colored the triangle that would be correct?

4. Homework – Similar Triangles WS

Lesson 4: Using Proportional Relationships

Lesson Objectives:
- Students will use ratios to make indirect measurements.
- Students will use scale drawings to solve problems.
- Students will make predictions and solve problems using proportional reasoning.

Target Grade Level:
- High school geometry

TEKS:
- G.11.A Similarity and the geometry of shape: use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.
- G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures.
- G.11.D Similarity and the geometry of shape: describe the effect on perimeter, area…when…dimensions of a figure are changed.

Embodied Materials Needed: Similar Triangles WS, Indirect Measurement WS, Meter sticks or tape measures

Observer Materials Needed: Similar Triangles WS, Indirect Measurement Packet, Rulers

Instructional Sequence:

1. Discuss homework – Similar Triangles WS

2. Activity 7 – Indirect Measurement
- *Embodied Version*
  1. Engagement – Ask students how they might measure something that was taller than they could reach with a measuring tape or ruler. For instance, how might they measure the flagpole in front of the school?
     a. Have students discuss with a partner.

b. Add that they have a mirror and a measuring tape as a resource.
c. Demonstrate how you can see the top of an object in the mirror by placing it on the ground between you and the object.
d. Have students discuss how they might use this information in finding the height of a tall object.

2. Have students work with a partner. Together they choose a tall object to measure. They use their bodies and the mirrors to form similar triangles and indirectly measure the height of the object.
    a. Students determine the minimum number of measurements they need.
    b. Direct Embodiment: Students go outside to collect the measurements.
    c. Students compute the height of the tall object.
3. Students work in their groups to complete parts 2 and 3 of the Indirect Measurement handout.

- *Observer Version*
    1. Engagement – Ask students how they might measure something that was taller than they could reach with a measuring tape or ruler. For instance, how might they measure the flagpole in front of the school?
        a. Have students discuss with a partner.
        b. Add that they could use a laser beam, a measuring tape, and a mirror.
        c. Show how the laser can reflect off of the mirror to hit the top of the building.
        d. Have students discuss how they might use this information in finding the height of a tall object.
    2. Have students work with a partner. Together they will find the height of the tall object drawn on their paper. The laser reflects off of the mirror to form similar triangles. They can indirectly measure the height of the object.
        a. Students determine the minimum number of measurements they need.
        b. Students use their ruler to collect the measurements. (or they are given by the teacher)
        c. Students compute the height of the tall object.
    3. Students work in their groups to complete parts 2 and 3 of the Indirect Measurement handout.

3. Activity 8 – What If?

- *Embodied Version*
    1. Display WhatIf PowerPoint.
    2. Have students work with a partner.
        a. Direct Embodiment: They will take a mirror and act out the scenario on the PowerPoint.
        b. For example, in scenario 1 asks, "What if the tall object were 2 times as tall, how much would you have to move to see the top of it."
        c. Discuss each scenario as a class.
- *Observer Version*

1. Display WhatIf PowerPoint.
2. Have students work with a partner.
   a. They will discuss each scenario on the PowerPoint.
   b. For example, in scenario 1 asks, "What if the tall object were 2 times as tall, how much would the laser have to move so it would hit the top of it."
   c. Discuss each scenario as a class.

4. Homework – Problem Solving WS 7-3

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